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OPTIMAL CURRENCY COMPOSITION OF THE FOREIGN RESERVES OF CHINA

By

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Abstract

Confronting the most serious economic crisis in recent decades together with all the countries in the world, China has done well as a representative of emerging economies as a new force to global economic growth. In 2009, China has successfully achieved the planned objective of the growth of GDP larger than 8%, which is 8.7% according to the data of National Bureau of Statistics of China. However, an important feature of the rapidly growing economy of China for quite a long time is its export orientation which leads to a huge amount of accumulated foreign reserves. In the financial crisis, foreign reverses also face larger exposure to more volatile foreign exchange market. In reaction to this, perhaps the optimization of the composition of foreign currencies of the foreign reserves can be one of the solutions to accelerate the process of economy recovery. Literatures manifest that the most popular method to tackle the problem of currency composition is the mean-variance portfolio management model, and this has also been proven to be actually used by many central banks. This framework will be applied in this paper as well. However, for the purpose of suiting the situation of China in the current time of financial crisis better, two points need to be emphasized especially. First, gold is included in the portfolio of reserves for the reasons of its international status and the recent rising price. In the modern international monetary system, multiple hard currencies, which are usually featured by floating exchange rate and freely convertible flexibility, exist simultaneously. As Renminbi or Chinese yuan is not one of them, China need these hard currencies to engage in international trade. However, during the current period of financial crisis which originated in the US, the status of dollar as to other major currencies is not as stable as before. Thus, the reason is enough to include gold into the optimized portfolio. Second, dependence measure of copula theory is used to replace the correlation between returns. This is for the reason that recent empirical studies reveal that different level of dependence of assets in financial market occurs in more volatile periods like the recent financial crisis, and common calculation of correlation cannot capture this change. Compared to the regular measure of dependence between random variables, correlation, the Kendall's Tau measures non-linear dependence which the correlation cannot. This paper based on the application of copula theory, is aiming to shed lights on the optimal currency composition for foreign reserves of China.

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1 Introduction

Problems about the management of foreign reserves can be generally organised into problems about the optimal quantity of foreign reserves and problems about the currency composition of the foreign reserves, or as in some literatures, questions on the 'absolute level' and the 'relative level' of foreign reserves.

The question on the absolute level of foreign reserves rose at a time when the World War II just ended. As a result of the lack of US dollar around the world, it is necessary for many countries to possess the amount of USD at least enough for the trade amount in three months. Later after the breakdown of the Bretton Woods system, various international currencies emerged and the pace for globalization fastened greatly. Some emerging economies began to accumulate large amount of foreign reserves for the purpose of preventing such financial crisis as the one happened in Southeast Asia in the later 1990s. How to determine the relative level of the foreign reserves has gradually become increasingly essential. China is one of these countries, and due to its export driven the economy growth type, it soon become the largest foreign currency holder in the world. Thus this question is of great importance for China.

There are mainly three methods to determine how the optimal composition of the foreign reserves of a country should be. They are transaction approach, mean-variance approach and the intervention-oriented approach. Dellas and Yoo (1991) provide evidence of the effectiveness of mean-variance analysis. Actual data from Korean central bank are utilised and the result shows consistency between mean-variance outcome and the actual composition. For China, the first two methods are more suitable, for the reason that China rarely uses foreign reserves to interfere the market. Papaioannou, Portes and Siourounis (2006) propose a dynamic mean-variance framework to deal with the question about the supposed international share of euro. This model calibrates the dynamic correlation and serial dependence in the variance-covariance matrix of currency returns by applying dynamic conditional correlation multivariate GARCH (DCC-GARCH). Thus it has the good ability to fulfil the task of optimal currency composition establishment.

However, the above analysis is still based on the traditional assumption of multivariate normal distribution. This is often not what happens in reality. Evidence show that in volatile periods, financial markets behave large deviation from what normal distribution predicts. Conclusions made using models under false assumptions of multivariate normal distribution would destroy their reliability and decisions built upon them may cause problems.

The recent financial crisis first triggered by the default of sub-prime mortgage in US has a worldwide influence and hit places outside America even harder. Some scholar suggests the large-scaled failure of financial system is only a superficial reason. The real problem lies in deeper in the structure of tangible economy. Since it seems recovery will be a long and difficult process, possibility of extreme events would be higher and hypothesis of normal distribution of returns would be harder to stand.

As a result, in addition to apply a mean-variance analysis with DCC-GARCH incorporated, a statistics tool, known as copula, is borrowed to cope with the problem of normal distribution deviation. Copula functions enable us to view the dependence structure of random variables and their marginal univariate distributions separately. More flexibility will allow for more accurate modelling of returns in volatile financial periods, and thus hopefully give us more reliable prediction. The present paper will use copula functions to deal with the estimation of variance-covariance matrix of currency returns in order to suggest a reasonable distribution of foreign reserves of China among different international currencies. But first, it is necessary to the framework method of this analysis – mean-variance analysis, and our modifications to the method for the purpose of suiting China.

The Markowitz mean-variance method is widely applied in the field of the optimal currency composition of foreign reserves. It diversifies risks by incorporating different assets' expected returns and their covariance. In order to modify it for analysis of currency composition of foreign reserves for China, choices on three aspects need to made, which are: first, currencies to be included in the portfolio and the reference currency; second, estimation

of asset return and variance-covariance matrix; and third, foreign reserves constraints on asset weights.

The choices on portfolio currencies and the reference currency are concerned with defining currency returns. Selecting portfolio assets means to choose the currencies which should be appropriate for the central banks to invest. The decision should be in consideration of the desirability for the currency in foreign trades and foreign debts as well as the currency's influence on foreign exchange market. Reference currency is a currency or an index used as a base to calculate the returns of each currency.

First, A typical portfolio of foreign currencies includes major international currencies, such as US dollar, pound sterling, Deutschemark and French franc or euro, Swiss franc, Japanese yen, etc.. However, there is a debate on whether should the gold be included in the context of today's monetary system. Ben-Bassat (1980) intends to simulate the optimal currency composition of the foreign reserves of bank of Israel and compare it with the actual data. The sample period is 1972-76, during which the international monetary system is undergone enormous changes – the fall of Bretton Woods system. Thus, gold is included as an important channel for risk diversification in context of greatly fluctuated foreign exchange market. However, seminal literatures in the later period, such as Rikkonen (1989), Dellas and Yoo (1991), Petursson (1996) and Papaioannou, Portes and Siourounis (2006), give up the inclusion of gold because they believe in the foreseeable future the current managed floating monetary system will still dominate, and under which gold lose its function of a venue for international payment. The present paper is written in a phase of global recovery from the most serious economic crisis in decades. Although there are already signs of the recession been casted away, actually there is still no clear pattern of the next round of economic growth. In view of this unpredictability, it is reasonable to include the gold in the portfolio from the perspective of a precautious central bank of China.

Second, the kinds of reference currency to represent the return documented the literature are native currency, U.S. dollar, import share weighted currency index or import currency share weighted index. They stand for different perspectives to explain the meaning of foreign

reserves. Ben-Bassat (1980) expresses the currency return in terms of dollar and import share index. Dollar is the dominant international currency, so the foreign reserves denominated by it represent its international value. The import share index is an import share weighted average of the currencies of corresponding nations. Currency returns denominated by it means a viewpoint of foreign reserves as a source for imports. Dellas and Yoo (1991) have a better data access and so use more accurately the import currency share as weight to replace the import share in calculation of the index. Native currency is used by PaPaioannou, Portes and Siourounis (2006) in their simulation for the BRICs (Brazil, Russia, India and China) to reflect the foreign reserves of each country in terms of their domestic value. The present paper adopts the first approach using dollar as reference currency.

In the mean-variance framework, estimations of variances and co-variances of returns from historical samples are needed to assess the diversification effect of a portfolio. Meanwhile, expected returns are also necessary for the assessment on the synthesized return of the portfolio. With respect to the estimation of expected returns, literatures suggest three assumptions, which are random walk, perfect foresight of the central bank and uncovered interest parity. The first presume the future change of exchange rate to follow a random walk so that the expected currency return is exclusive for the effect of fluctuation of exchange rate. Perfect foresight assumes the central bank has an information advantage to predict return which can allow the substitution of the expected return by the actual return of next term. Uncovered interest parity is the most popular theory to dictate the expect return of exchange rates by its relationship with the interest rates of two countries.

In the aspect of estimation of variance-covariance matrix between currencies, methods documented in literatures vary from simple independent identical distributed historical sample calculation to more complicated dynamic conditional correlation multivariate GARCH. However, Fantazzini (2004) points out three shortcomings of the correlation as a measure of variable dependence. First, the correlation only exists when the variance is finite. Simple bivariate t-distribution with two degree of freedom will not fit. Second, correlation only reflects linear dependency so unless the joint distribution is normal zero correlation does not mean independent variables. Third, sometimes when monotone transformation is needed for the measure of dependency between variables, correlation fails to stay invariant. The

present paper attempts to utilise Kendall's Tau and Tail dependence based on copula calibration to measure interdependency of the currency returns.

In addition, some constraints should be placed on the investment weights among different currencies because the management of foreign reserves should not only satisfy needs for optimising risk and return, but also complete the mission of ensuring foreign debt payments and international transaction payments. Therefore, it is necessary for China to preserve certain proportions of currencies of such countries which are actively engaging financing and trading activities with China.

The following parts are arranged in this way. Section 2 is a literature review on development of currency composition of foreign reserves, copula functions and their application. Section 3 states methodology and data processing. Section 4 will present results of composition with analysis. Section 5 makes conclusion.

2 Literature review

For any kind of management, it is of priority to determine what the final goal is and find out the mechanism the system in which objects to be managed runs. It is no exception to manage the relative level of foreign reserves of China. Thus, we first review literatures about the motivations behind the behaviour of foreign reserves accumulation and attempt to classify what type China belongs to. Then appropriate method for managing can be decided.

2.1 Motives for the management of foreign reserves

Roger (1993) argues three main incentives for holding foreign reserves by the central bank. The motives are for transaction, for precautious reasons and for portfolio considerations.

Transaction motives

Foreign exchange reserves are held for the reason to finance the payment of foreign transactions. This kind of demand is not so high in developed countries, as private sectors in these countries possess adequate access to foreign finance market. Efficient international capital market ensures best price to finance the need for transaction. However, people in developing countries with limited access to international capital market may find the foreign reserves more useful, especially when people live in a country where extensive controls over foreign exchange are imposed. In this case, financing from reserves held by the central bank will cost less than borrowing to finance currency account deficit.

Intervention needs

The intervention needs for holding foreign exchange reserves are referred to the motives for central banks to prevent or amplify the changes in international exchange market to their domestic economies. This kind of demand is especially significant for countries with a very

open goods and capital market or countries with a peg currency or a peg basket of currencies. Insofar as the duration and magnitude these interventions can be categorized into two, short-termed sterilized intervention and medium-termed unsterilized intervention.

Short-termed sterilized intervention

This motive is identified as the most important reason for developed countries, which are with open goods and capital markets, to use the foreign reserves. The intervention is launched in a very short period and in a sterilized fashion, which means in a fashion the money supply will not be affected, to offset the capital movement caused by speculative investment behaviour. Thus the economic "bubble" is prevented. Alternatively, short-termed intervention is needed within countries with changes of their currencies committed to be restricted in a range with each other, like ERM participants in the period before the launch of euro. This measure is to buy the country participant time to adjust their domestic policies to achieve their goal of exchange rate.

Medium-termed unsterilized intervention

In contrast to the sterilized intervention, medium-termed unsterilized intervention refers to large magnitude usage of foreign reserves and changes of money supply as a consequence in order to maintain stable domestic economy. This sort of measure is mostly utilized by developing countries due to lack of well-developed domestic financial market. Essentially, for the purpose of achieving goals in terms of interest rate and exchange rate, three alternative measures can apply, adjusting of domestic financial policy, using large amounts of foreign reserves and allowing the exchange rate to float. There is extensive literatures support that the optimal method is to change the domestic policy in a well-developed and international market integrated domestic market. The floating exchange rate method lies somewhere between the other two. However, for a country with a domestic market with insufficient efficiency, needs for unsterilized intervention are significant, even more evident when in a country where the monetary authority is actually the currency board.

Return and risk consideration

Wealth diversification considerations mainly concern decisions about the currency composition of the foreign exchange reserves. The return and risk considerations rarely affects the quantity of the foreign reserves. However, in the case of countries holding huge volume of reserves, since the quantity far exceeding the requirement for transaction and intervention motives, such countries like China, there leaves great space for return and risk considerations. Many literatures argue the current amount of foreign reserves of China has far surpassed its optimal level, such as Wu Lihua and Li Peng (2006) and Shi Jianren and Guo Encai (2001). Two exceptions where the wealth diversification takes effects on the quantity level of foreign reserves are: first, central take responsibility for the foreign exposure; and the second is the public sector substitute the private sectors for holding foreign reserves. In the first case, in order to enhance the credibility of a country, gross reserves may be increased if more foreign debts are issued. The second case refer to foreign exchange controls being imposed on the private sector, like China, and the central bank become a agent for the private assets of foreign currency. Therefore, in this case considerations about the tradeoff between return and risk are especially essential.

In response to three types of motives for holding foreign reserves, Zhichao Zhang and Yonglin Xu (2010) summarises there are three types of methods for managing currency composition. Intervention method is not suitable for China, since China rarely directly use foreign reserves to interfere market. A representative of transaction method is by Dooley. Dooley (1987) maintains the management of currency composition of foreign reserves of a country's central bank should focus on total foreign assets and liabilities rather than just foreign reserves. This kind of adjustment on both sides of assets and liabilities need much coordination among national institutions of a country. It is with difficulty to realise and it has few followers on this direction.

The most popular method for currency composition of foreign reserves is the mean-variance analysis in portfolio theory. This method is motivated by the return-risk consideration.

Through analysing the dependence structure of various currency returns, it aims to diversify risk by investment weight assignment in a currency portfolio. The development of this method is reviewed in the following.

2.2 Development on methods motivated by return-risk motive

With respect to researches on currency composition of foreign reserves, there is relative little contribution in the area of empirical study. This is partly due to the large difficulty of getting access to the data of a specific country. In the region of relative level of foreign reserves, it is more often to see normative researches, which is to tackle how the composition should be rather than proving factors of the composition in real world by analysing data.

Ben-Bassat (1980) is the first to consider the currency composition in a Markowitz mean-variance manner. His work stands on a position for the central bank of Israel, which he considers as a representative for a semi-industrial country, and he wants to find out whether profit consideration plays an important role when making decisions about the currency composition for foreign reserves in the semi-industrial and developing countries. A comparison of the simulated optimal portfolio with the actual data from the central bank of Israel is made, and the author's hypothesis is confirmed by the conclusion. The research methods involved in the study first includes an analytical approach to specify the different motives for holding reserves by developing countries and developed countries. Developing countries hold a larger amount of foreign reserves than the amount needed to satisfy the motive for intervene the foreign exchange market, so the profit objective is a major consideration. In the conduct of mean-variance analysis, both dollar and import weighted index are used as reference currency representing two views of the foreign reserves. When determining an optimal portfolio from a set of portfolios along the efficient frontier, CAPM is assumed instead of make assumption of utility function. The Sharpe ratio is applied and the risk-free interest rate is calculated by assign import share as weights to the T-bills of different countries.

The study of Ben-Bassat (1980) is a pioneer research at the beginning of a new international monetary system of managed floating exchange rate. Previous studies on international portfolio mainly focus on choices for individuals or firms. Kenen (1967), Officer and Willett (1969), Hagemann (1969), Steckler and Pickarz (1970) and Makin (1971) study the currency composition of foreign reserves of a country, however, only restricted in a two-option choice between gold and dollar, because at that time gold dominated in the monetary system and the shift from fixed to floating exchange rates had not happened. In the floating exchange rates international monetary system, dollar lost its special place as representative of other currencies, a portfolio which includes a whole array of currencies is needed.

Brown et al. (1986), Dellas (1989) and Heller and Knight (1978) also use the mean-variance method to deal with the composition question, but their results are compromised due to lack of access to the actual data of a specific country. Dellas and Yoo (1991) make a contribution to this kind of deficiency. They for the first time use several sets of actual data, for example, import currency shares and portfolio of foreign reserves of Korea. Besides, their study also attempts to determine a better model for optimal currency composition of foreign reserves between the mean-variance model and the consumption capital asset pricing model (CCAPM). They conclude that the similar result of the portfolio by mean-variance model with the actual data of the central bank of Korea indicate potential practice of mean-variance by central banks. In contrast, the test for CCAPM is inconclusive due to lack of sensitivity of the test to coefficient changes. Dellas and Yoo (1991) improve the importance of mean-variance method greatly in the area of currency composition of foreign reserves. With regards to the specifics of the mean-variance methods involved in this paper, one contribution is that they bring out the difference between using the import share and the import currency share as weights to calculate the reference currency. Two results differ largely, which means that in order to be accurate data on import currencies is strongly advised. Furthermore, in comparison with the actual portfolio, an insightful way to choose optimum from the efficient frontier is invented. They select one with the same variance with the actual portfolio. This measure can not only reflect the level of risk requirement of the central bank, but also provide an intuitional approach for comparison.

Petusson (1996) use the Markowtiz mean-variance portfolio management theory to find the optimum composition of foreign reserves of Iceland. The contributions to the method include the application of ARMA to estimate the expected return and the use of utility function to determine optimum solution. The attempt to replace a simple random walk assumption for the changes of exchange rate by ARMA model is a significant leap towards reality. Although the method is still relatively simple, proof of superiority to the random walk model is made by the author. Utility function is one of the foundations of portfolio theory, the incorporation of the utility function, which hence enables the direct adjustment of the coefficients in the utility experience, makes the model more flexible.

Papaioannou, Portes and Siourounis (2006) conduct a research on one of the most important issue in the international monetary system in present time: impacts of the rise of euro in the world on the composition of international foreign reserves. They attempts to discover if there is a strong force for the international foreign reserves to diversify away from dollar, as a result of its problem of huge fiscal deficit and, in the meanwhile, other currencies' increase of liquidity. There is a debate among academics that whether central banks are facing a "concentration risk" of their foreign reserves on the US dollar. Some argue there are severe challenges for dollar, while others contend that the most reserves accumulated countries still expect to maintain their economic growth by an export-oriented way, with the US as their main destination, and so their exchange rates to dollar should be ensured stable. If factors for US dollar to rapidly lose its dominance to, say, the euro are solid, dollar will depreciate and a significant change will occur to the international finance. The research possesses realistic significance. However, the conclusion manifests that dollar would maintain high percentage of international reserves in the optimizer, and euro will dominate only if more countries to adopt euro-based anchor currency policy.

In the perspective of method, an advanced dynamic mean-variance optimisation model is applied. The paper of Papaioannou, Portes and Siourounis (2006) is one of the most comprehensive analyses using the mean-variance model to optimal composition question. With regards to the estimation of expected return, all of the three popular assumptions are used, which are random walk, perfect foresight and the uncovered interest parity. In addition, transaction cost is incorporated by the bid-ask gap as a proxy. The model is called dynamic

because besides the regular simple historical estimation, constant conditional correlation multivariate GARCH (CCC-GARCH), and Dynamic conditional correlation multivariate GARCH (DCC-GARCH) are utilised to estimate the variance-covariance matrix of currency returns. With respect to the selection of optimum from a set of portfolio along the efficient frontier, value at risk (VaR) constraints are cited and they restrict the risk preference of central banks to be 3.3%. First, aggregated data at global level is used and a representative central bank is assumed to analyse the international status of dollar, euro and other major currencies. After that, since developing countries are of great significance considering their quantity of reserves, nation-specific analyses are conducted for central banks of Brazil, Russia, India and China (the BRICs). In the analysis, additional restrictions are set according to international trade and foreign debt volume in main currencies, and are imposed on currency shares in the optimal portfolio. The results can be summarized as: First, duo to high volatility of interest rate and exchange rate, the optimal allocation of foreign reserves is very unstable. That is maybe why central banks proceed slowly when diversifying. Second, the US dollar takes great account in the optimizer, about 60%~65%. Third, evidence shows the lower share of euro in the optimal portfolio than in reality. This means the euro's international stance is appreciated and the surplus to the optimizer comes from other main currencies like pound sterling, yen and the Swiss franc rather than the dollar. Fourth, the anchor currency to which most developing countries' currencies is pegged plays the most influential role in determining the dominant international currency. If more countries move away from dollar and adopt euro-based anchors, that will poise a difficult challenge to dollar.

Finally, Papaioannou, Portes and Siourounis (2006) recommend further research directions in three aspects. With regards to how to enhance international status of euro, in addition to encourage more countries to treat euro as their peg currency, central banks' allowing short sell in their management of foreign reserves could boost euro. With respect to which method is appropriate for analysis of currency composition, authors maintain mean-variance model with modification to suit specific demands of a particular central bank is preferred. As to extend the framework further and deeper about management of foreign reserves, investment instruments like government bonds, commercial paper and equity of main industrial countries should be incorporated and risk management can be considered.

Through review on these literatures, one-period mean-variance method has been developed quite well. Further breakthroughs in methodology can be made either by establishing multi-period MV model or by better estimation of expect returns and covariance in order to conquer the assumption of multivariate normal distribution. Copula is such a devised to tackle the later problem.

2.3 Literatures on copula theory

Limitation of Pearson's correlation to capture dependence among random variables is the reason for the necessity of copula. Correlation coefficient is a proper measure of dependence within the setting of multivariate normal distribution. However, there remain some fundamental drawbacks of this kind of dependence measurement, especially if assets are not jointly normally distributed. The drawbacks include (Fantazzini, 2004): first, the variance of the distribution must be finite. This limitation is so strict that a simple bivariate t-distribution would violate. Second, independent random variables have zero correlation coefficient. However, the converse is not necessarily true, because correlation reflects only linear association only in limited examples the latter is established such as in jointed normal distribution. A simple case where the converse is not true is when the marginals are Gaussian while their joint is not normal. Third, correlation is not invariant to strictly monotone transforms. This argument will be explained after the introduction of copula later in this paper, which allows us a new perspective to view the marginal distributions and their joint association separately. Briefly, the third shortcoming means correlation again limits the number of possible types of distributions. In general, the traditional measure of dependence correlation coefficient seems inappropriate in volatile financial market conditions where assets behave apart from normality.

Dated back as far as Mills (1927), since then evidences have been documented that many common financial variables are distributed non-normally. One of the most important benefits of copula is that it allows the identification of special marginal distributions to be separated from the dependence structure of random variables. This does not only give us the freedom to exploit univariate techniques individually and then combine them with dependence measures,

but also provide us the ability to measure the association to more varieties of distributions. Kendall's tau, for example, measures concordance between two random variables. Concordance means large value of one random variable coincides with large value of the other, if not so, they are discordant. Tau avoids failure to capture non-linear association and possesses most desired property for a dependence measure between variables. In the following, we will make a basic introduction to concepts in copula theory.

Definition of copula: the copula of $(X_1, \dots, X_n)$, where $X_1 \sim F_1, \dots, X_n \sim F_n$, and $F_1 \dots F_n$ are continuous, is the joint distribution function of $U_1 \equiv F_1(X_1) \dots U_n \equiv F_n(X_n)$. (Nelson, 2006)

The variables are known as the 'probability integral transformations' of $X_1, \dots, X_n$. Fisher (1932) discovered that through the transformation of $U_i = F_i(X_i)$, X_i becomes a uniform distribution U_i regardless whatever the distribution of random variable. This definition that it is the joint distribution of uniformly distributed random variables means copula functions must have following properties:

Properties of a copula (Patton, 2001): An n-dimensional copula has the following properties:

1. The range of $C(u_1, \dots, u_n)$ is the unit interval [0,1]

2. $C(u_1, \dots, u_n) = 0$ if any $u_i = 0$ for $i = 1,2, \dots, n$.

3. $C(1, \dots,1, u_i, 1, \dots,1) = u_i$, for all $u_i \in [0,1]$

4. $C(u_1, \dots, u_n)$ is n-increasing, which means:

$$\Delta C_a^b = \sum_{(\varepsilon_1,\dots,\varepsilon_n)\in(0.1)^n} (-1)^{\sum_{i=1}^n \varepsilon_i} C(\varepsilon_1 a_1 + (1-\varepsilon_1)b_1, \dots, \varepsilon_n a_n + (1-\varepsilon_n)b_n) \geq 0 \qquad (1)$$

One attraction of the utilization of copula is it displays clearly the information of variable dependencies in a joint distribution, excluding the interference of marginal distributions of individual random variables. In the usual sense, a joint distribution function is assumed to contain all the information of the probability for random variables, margins and dependencies.

Since the transformation of $U_i = F_i(X_i)$ can turn variables of whatever kind of distribution into uniformly distributed variables, thus copula functions, the final result of the probability integral transform, is like a filter, manifesting the dependence structure of variables only. This is also the reason why copula is also called the 'dependence function' (Galambos 1978).

Another reason for the importance of copula in expressing the dependence structure for financial modelling and simulation is the unique corresponding relation between copula and general distribution functions. This is given by Sklar (1959). Sklar's theorem is the main result in copula theory.

Sklar's theorem for continuous distributions: Let H denote a n-dimensional distribution function with margins $F_1, \dots, F_n$. If assume that $F_1 \dots F_n$ are continuous, then there exists a unique copula C such that

$$H(x_1 \dots x_n) = C(F_1(x_1), \dots, F_n(x_n)) \tag{2}$$

Conversely, if $F_1(x_1), \dots, F_n(x_n)$ are distribution functions and C is a copula, then $H(x_1 \dots x_n)$ defined by the above function is a joint distribution of $(X_1 \dots X_n)$ with each margin of . $X_1 \sim F_1(x_1), \dots, X_n \sim F_n(x_n)$

In fact, the converse direction of the Sklar's theorem has caused greater interests of academics. The reason is the theorem means margins of any distribution can be linked with any type of copula function to form a joint distribution with desired properties and meanwhile with no loss of tractability. This fills the gap in the before statistics literatures that while enormous types of univariate parametric distributions are established, there are much fewer multivariate parametric distributions. Calibration and simulation with multivariate parametric distributions improves our ability to analyse financial data.

In the further development, the question of how to generate parametric copula functions becomes critical. Basically, there are two ways. The following is the first way. It allows generating copulas from each existing multivariate joint distribution.

Approach 1: Let H be any joint distribution of n random variables with continuous marginal distributions $F_1, \dots, F_n$. Let $F_1^{(-1)}, \dots, F_n^{(-1)}$ be the (quasi-) inverse of the margins. Then there exists a unique copula $C: [0,1] \times \dots \times [0,1] \to [0,1]$ such that

$$C(u_1, \dots, u_n) = H(F_1^{(-1)}(u_1), \dots, F_n^{(-1)}(u_n)) \quad (3)$$

Given any joint distributions we can get a copula by above corollary. Typical examples include elliptical copulas such as normal copula from multivariate normal distribution and t-student copula from multivariate t-distribution. Normal copulas and t copulas have the dependence structures of normal and t distribution, but with the flexibility to combine with any kind of univariate distribution to generate a new joint distribution.

The second way to form a copula is by using a strictly decreasing convex function as a so called generator function. This type of copula is known as Archimedean copulas.

Approach 2 (Archimedean copula): Let $\varphi: [0,1] \to [0, \infty]$ be a function which is continuous, strictly decreasing and convex. The function $C: [0,1]^2 \to [0,1]$ defined as

$$C(u_1, u_2) = \varphi^{-1}[\varphi(u_1) + \varphi(u_2)] \quad (4)$$

is an Archimedean copula and φ is known as the generator of the copula. This type of copula generation enables us to produce vast amount of copulas because there are a great number of function φ. Together with the ability to link a good deal of univarite marginal distributions, it gives unprecedented flexibility to simulate various phenomenons in financial markets. Common examples of Archimedean copulas are Gumbel copula and Clayton copula (for one-parameter and multi-parameter Archimedean copulas see Nelson, 2006).

With respect to estimation for copulas, there are three types of method (Nelson, 2006). Maximum likelihood method is standard and is able to find the consistent estimator with largest likelihood function. A second approach for estimation is the inference functions for margins method (IFM – or two stage method). This approach can significantly decrease

computational effort. However, it may lose some optimality compared to ML method. The third is the canonical maximum likelihood (CML) method. It differs from the IFM because its estimation process is performed without assumption about the parametric form of the marginal distributions. Thus, CML can avoid the problem of fault estimation of margin parameters.

The final preparation of copula knowledge for its incorporation in mean-variance framework is the concept of Kendall's tau. The demand for a number, a metric, to represent the extent of dependence between random variables is seen everywhere in statistics application fields. Conventionally, Pearson's correlation plays this role. In portfolio management theory, investors assign more weight in assets whose returns are with lower correlations so that diversification effect would lower the total risk of investment. However, this approach is only appropriate if assets returns are in joint multivariate elliptical distributions (see Alexander, 2008). Conversely, evidences show it is often not the case for elliptical distributions, especially when we analysing high frequency financial data. Dependence measures known as rank correlations are invented in order to avoid the above problems. Typical one of them is Kendall's Tau. It is a metric of dependence built on the concordance between variables, and hence surpasses Pearson's correlation in not just reflecting the linear associations (see Nelson 2006). Kendall's tau can be considered as a measure of the monotonic dependence between random variables, whereas Pearson's correlation only a measure of linear dependence. This is the reason why it can overcome the defects we mentioned above about the correlation (Alexander, 2008).

3 Methodology and Data Processing

3.1 Objective function

For any investment decision in financial markets, it is always about a trade-off between return and risk. In terms of the structure of currency composition of foreign reserves, the adjustable factor is the share distributed to different currencies on considerations of the risk and return characteristics of each combination. Thus it is an analogical problem of portfolio management. When returns are substituted by future expected means and risks by variances, the objective function to optimise the currency composition of foreign reserves of China within the framework of a single-period mean-variance analysis is to minimize the risk subject to a designated return. That is because from the central bank's perspective, risk management is more sensible an objective than profitability.

$$min\sigma_{R,t+1}^2 = W_t' \Omega_{t+1} W_t \tag{5}$$

Subject to:

$$E(R_{t+1}) = W_t' \boldsymbol{\mu}_{t+1} \tag{6}$$

$$\sum_{i=1}^{n} w_{it} = 1\,; \quad w_{it} \geq 0; i = 1,2,\dots,n \tag{7}$$

where W_t is the weight vector in the period t, with its element of w_{it}, i = 1,2,…,n, representing the weight in the portfolio of currency i; $\boldsymbol{\mu}_{t+1}$ is the expected mean vector consisting $\mu_{i,t+1}$, i =1,2,…,n; R_{t+1} is the sum of $r_{i,t+1}$ representing the return of portfolio in period t+1; Ω_{t+1} is expected variance-covariance matrix of each currency asset; and $\sigma_{R,t+1}^2$ is the variance of the portfolio which is the target for minimization in the model.

The expected variance-covariance matrix Ω_{t+1} in the application of Pearson's correlation coefficient can be written as:

$$\begin{bmatrix} \sigma_1^2 & \cdots \rho_{1i} * \sigma_1 * \sigma_i \cdots & \rho_{1n} * \sigma_1 * \sigma_n \\ \vdots & \ddots & \vdots \\ \rho_{n1} * \sigma_n * \sigma_1 & \cdots \rho_{ni} * \sigma_n * \sigma_i \cdots & \sigma_n^2 \end{bmatrix} \quad (8)$$

where ρ is correlation coefficient. This model is effective under the assumption that currency returns are distributed under multivariate normal distribution. However, in the context of copula calibrated model, Pearson's correlation ρ is substituted by another measure of variable association, Kendall's Tau.

$$\begin{bmatrix} \sigma_1^2 & \cdots \tau_{1i} * \sigma_1 * \sigma_i \cdots & \tau_{1n} * \sigma_1 * \sigma_n \\ \vdots & \ddots & \vdots \\ \tau_{n1} * \sigma_n * \sigma_1 & \cdots \tau_{ni} * \sigma_n * \sigma_i \cdots & \sigma_n^2 \end{bmatrix} \quad (9)$$

where τ is the coefficient of Kendall's Tau. The calculation of this coefficient is based on different specification of three types of copulas, normal copula, t-copula and Gumbel copula. A five-year daily data window is utilized for estimation. The allocation of foreign reserves is optimised through both assumptions and a comparison is presented.

After the efficient frontier is prepared, it is critical to find the optimal portfolio according to the investor's risk preference. In the present paper, the central bank of China is considered more conservative than average investors. When we set the US dollar as a risk free investment with its interest rate as the risk free rate, Sharpe ratio can be used to decide the satisfactory efficient portfolio with the help of risk preference coefficient ranging from 2 to 4. The coefficient represents the degree of risk averseness, with 4 as the most conservative while 2 as the most aggressive. We assume 3.3 of China's risk acceptance level.

3.2 The selection of currencies in the portfolio

Roger (1993) concludes in his review about literatures on the management of foreign reserves that the major functions for foreign reserves to exist can be summarised to three points. They are for financing transactions, for intervening foreign exchange market and for preventing financial sudden shocks. Several seminar empirical researches also indicate the transaction function and precautious function are factors actually affecting the structure of international currency reserves (Heller and Knight, 1978; Dooley, 1986; Dooley, Lizondo, and Mathieson, 1989; Eichengreen and Mathieson, 2000; Dooley, Folkerts-Landau, and Garber, 2003; and Xu and Zhang, 2010). Thus, it is necessary for the selected portfolio to fulfil such basic functions as well as to provide a sufficient range for risk diversification.

In 2005, China altered the policy of foreign exchange rate of Chinese Yuan Renminbi from pegging US dollar to by reference to a basket of currencies, shown as follow table.

Table 1

RMB official currency basket		
	Symbol	Name
Main Currencies	USD	U.S. Dollar
	EUR	Euro
	JPY	Japanese Yen
	KRW	Korean Won
Secondary Currencies	SGD	Singapore Dollar
	MYR	Malaysian Ringgit
	AUD	Australian Dollar
	CAD	Canadian Dollar
	RUB	Russian Rouble
	THB	Thai Baht

This supplies us the information about major currencies China is using for satisfying the basic functions of foreign reserves. Therefore our portfolio should include these ten currencies.

There are other currencies which should be included for the reason of diversification. By the means of making a matrix of correlation coefficient between 20 major currencies based on simple historical exchange rates changes, portfolio candidates with better diversification effects can be determined. The matrix is shown in the appendix including the detailed selection criteria. As the result of the analysis of the matrix, in addition to the ten currencies in the pegged basket, UK Pound (GBP) and Swiss Franc (CHF) should be included as well.

Gold has long been in a significant position in the international monetary system. However, since the break down of the Bretton Woods system, gold has lost its ability in international trade, and gradually fades out of foreign reserves in many countries. But the significance of gold should not be neglected. Especially in the recent time of worst financial crisis and gentle recovery, huge deficit of US has amounts great pressure for the dollar to depreciate. As an important venue for diversification and established symbol of solid value, gold should be involved in the portfolio for analysis.

3.3 Expected returns

In the objective function articulated in the previous section, in order to solve the optimal solution, two parameters should be estimated first. One is expected returns of currencies in the end of the investment period and the other is the variance-covariance matrix to reflect correlations among them. In this and next sections, methods for estimation of these two parameters are stated.

It will always be a guess, in the context of social science, a statistical estimation, to predict the future. In terms of the anticipation of future return for holding foreign currencies, several theories are claimed to be reasonable. The most cited theory to forecast the foreign exchange rate is the uncovered interest parity (UIP from now on). It is a similar version to the pricing theory of forward contract in the exchange market, named covered interest parity (CIP). CIP dictates the change of the future exchange rate between two currencies stated on a forward contract should be completely compensate the difference of the riskless interest rates of these

two countries in the same period, otherwise, arbitrage would happen. UIP maintains that the expected future spot exchange rate should equal to the forward price, because arbitrage opportunity should be eliminated if the market is efficient enough. However, UIP does not always hold in reality. Literatures on currency composition of foreign reserves often make another assumption that the central banks possess perfect foresight about the expect return due to its information superiority. In this case, actual changes are used to represent the estimation of central banks. Neither of these two theories is satisfactory to simulate the reality. Since no theory can predict perfectly, some argue the exchange rate may actually just follow a random walk. These three propositions are all attempted in this paper. And for the purpose of incorporating the influence of transaction costs, ask-bid disparity under UIP is raised as the fourth assumption for expected returns. The formula of expected currency return in general is:

$$E(r_{i,t}) = b_{i,t} + E(s_{i,t} - s_{i,t-1}) \tag{10}$$

where the expected currency return of currency i at the time t, $r_{i,t}$, equals to the sum of interest rate of the currency for the period between t-1 and t, $b_{i,t}$, and the expected difference of log exchange rate between time t, and time t-1. When investing in gold, it is assumed the native interest rate of China is earned. Interest rate of a currency $b_{i,t}$ is known at the beginning of an investment period, but the latter part of right side of the equation needs some assumptions.

The first assumption of the expected return is the FX rate of a currency follows a random walk, and the exchange return is a stationary process. As a result, the expectation of exchange return is zero, leaving just $b_{i,t}$ standing for the expected currency return under the random walk assumption. Second, perfect foresight assumption means central bank of China can precisely predict the expected exchange rate of next period. Thus, the actual realizations of FX rates are used and the formula becomes:

$$E(r_{i,t}) = b_{i,t} + s_{i,t} - s_{i,t-1} \tag{11}$$

This assumption is sensible on the consideration that central banks possess advantage on market information. Under the uncovered interest parity (UIP) assumption, all currency returns equalise with each other.

To deal with the uncertainty further, an error term is appended for each of the three methods for expected return calculation. The error terms are simulated five hundred times by Gaussian distributions with zero mean and the standard deviation of each currency returns. The mean of the 500 repeats is reports as the estimation for the currency return.

3.4 Variance-covariance matrix

The unstable correlations among currencies returns are mostly criticized as a drawback of the mean-variance approach. Even a small change in the variance-covariance matrix will confer a significant disturbance in the final shares. To address this challenge better, this paper attempts to include more models to describe the correlations. In addition, for the purpose of capturing features of volatility dependence traditional linear correlation coefficient fails to depict, several copula functions about different distribution functions of financial assets are proposed and two measures of dependence are raised, Kendall's tau and tail dependence.

I. Simple Historical

The conditional correlation between two random variables is defined as follows, if the random variables and each have zero mean (if not the transformation is easy and without large impact to further estimation), the correlation should be:

$$\rho_{12,t} = \frac{E_{t-1}(r_{1,t}r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2)E_{t-1}(r_{2,t}^2)}} \tag{12}$$

This definition limits the result of correlation lies within the interval [-1, 1]. This is the full sample case. In reality, estimator based on sample observations must be attempted since it is simpler and usually the population is assumed to be infinite and inclusion of full sample is not possible.

The simple historic estimator assumes no relations among the time series random variables, i.e. the currency returns today and yesterday are independently and identically distributed (iid). In that case, the estimator of two variables is:

$$\hat{\rho}_{12,t} = \frac{\sum_{s=t-n-1}^{t-1} r_{1,s} r_{2,s}}{\sqrt{(\sum_{s=t-n-1}^{t-1} r_{1,s}^2)(\sum_{s=t-n-1}^{t-1} r_{2,s}^2)}} \quad (13)$$

II. Copulas: Kendall's tau

In order to calculate a population Kendall's tau and to put it into variance-covariance matrix we modified in previous paragraphs, copula distributions must be calibrated from the sample first. Two types of multivariate copulas are attempted, which are the Gaussian copula and the t copula.

The CDF function of Gaussian copula is

$$C(u_1, \dots, u_n | \mathbf{\Sigma}) = \mathbf{\Phi}(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n)) \quad (14)$$

where is $\mathbf{\Phi}$ the standard multivariate normal distribution with correlation matrix $\mathbf{\Sigma}$. Φ^{-1} is the inverse of the univaraite standard normal distribution.

For the t-copula the CDF function is:

$$C_v(u_1, \dots, u_n|\mathbf{\Sigma}) = \mathbf{t_v}\left(t_v^{-1}(u_1), \dots, t_v^{-1}(u_n)\right) \quad (15)$$

where besides the similar parameter as the Gaussian copula, the correlation matrix $\mathbf{\Sigma}$, an additional parameter, the degree of freedom, v, needs also to be estimated. $\mathbf{t_v}$ and t_v are respectively the multivariate and univariate t distributions with v degree of freedom. The calibration of these two copulas is conducted by the method of Maximum Likelihood. Empirical univariate CDF for each currency returns are executed for the estimation of marginal distributions.

Kendall's tau is defined on the basis of concordance. According to the calibrated copula function, the population version of Kendall's tau can be calculated according to the following formula:

$$\tau(X, Y) = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1 \quad (16)$$

3.5 Constraints by trades and foreign debts

Mean-variance analysis is a method which emphasizes the efficiency of the investment in terms of return and risk. However, to manage the currency composition of the foreign reserves is not just to take care of an investment. Foreign reserves of a country must first ensure normal international business and finance activities. Therefore, we make two types of optimal portfolio restricted by data of foreign debt partners of China and foreign trade partners of China respectively. The currency weight constraints on main foreign debt issuing currencies is set to be 50% of the foreign debt share of that currency. (Data of currency composition of foreign debt from World Bank's Global Development Finance Database) To be comparable the constraints corresponding to the trade volume is also set to be 50% of the

share of the currency in terms of the sum of import and export. (Data of trade direction from Direction of Trade database of IMF). Proportions of foreign debt and foreign trade of the countries selected in the portfolio with China are reported in the following tables.

Table 2

FOREIGN DEBT PARTNERS	
	2008
USD	81.68%
Euro	6.62%
JPY	9.14%
GBP	0.04%
CHF	0.03%
Mutiple Currencies	2.35%
All Other Countries	0.12%

Source: Global Development Finance Database of the World Bank

Table 3

FOREIGN TRADE PARTNERS	
	2008
AUSTRALIA	2.29%
CANADA	1.35%
EURO AREA (EXC. UK)	11.13%
JAPAN	10.43%
KOREA, REPUBLIC OF	7.27%
MALAYSIA	2.09%
RUSSIA	2.22%
SINGAPORE	2.05%
SWITZERLAND	0.44%
THAILAND	1.61%
UK	1.78%
US	13.06%

Source: Direction of Trade Database of IMF

3.6 Data characteristics

The entire sample analysed spans from Jan 01, 2004 to Dec 31, 2008 for five years. Thus the objective of optimization of the currency composition is to give advice on the beginning day of 2009. This one year available data not being used is designed for the purpose of further examination of the effectiveness of the current method. Daily data is used to estimate the changes of the currencies' returns, and a five-year window right prior to the beginning of the analysing year is employed to form the variance-covariance matrices.

The interest rates of various countries come from Thomson Datastream on a basis of five days a week, which means the data are only recorded in working days. Three months' interbank rate is used to represent the interest rate for carrying the reserves of a country. The history data of exchange rates come from OANDA Corporation, and trimmed to fit with the five days working days format to be same with the interest rate data. The followed empirical experiments are reported based on these data.

4 Results and analysis

Since the most significant improvement of the present paper to the way to managing China's currency composition of foreign reserves is the incorporation of copula functions, the estimation result is reported in the order of three different types of variance-covariance matrices (VCM).

As to the simple historical estimation, it refers to the assumption that the FX daily changes are independently identically distributed (iid.) and the variables confine to multivariate standard normal distribution. The result is summarized in the following table.

Table 4

Jan 2009	Simple Historical					
	Foreign Debt Constraints			Foreign Trade Constraints		
	RW	PF	UIP	RW	PF	UIP
USD	40.84%	40.84%		6.53%	6.53%	
EUR	3.31%	3.31%	3.31%	6.45%	6.45%	6.45%
JPY	4.57%	4.57%	5.18%	5.21%	5.21%	5.86%
KRW	28.20%	4.93%	1.18%	56.87%	42.82%	3.63%
SGD	1.18%	1.18%	30.49%	1.02%	1.02%	27.46%
MYR	1.18%	1.18%	1.89%	1.04%	1.04%	1.04%
AUD	1.18%	1.18%	1.18%	1.14%	1.14%	1.14%
CAD	1.18%	1.18%	1.18%	0.67%	0.67%	0.67%
RUB	1.18%	1.18%	41.67%	19.13%	1.11%	41.08%
THB	1.18%	1.18%	10.27%	0.80%	0.80%	10.20%
GBP	1.18%	1.18%	1.18%	0.89%	32.96%	0.89%
CHF	14.85%	1.18%	1.18%	0.22%	0.22%	0.22%
GLD	0.00%	36.94%	1.31%	0.00%	0.00%	1.34%
RW=Random Walk		PF=Perfect foresight		UIP=Uncovered Interest Parity		

To break the naive assumption of multivariate normal distribution, two types of copulas are incorporated, which are normal copula and t copula. In order to achieve ideal effect of diversification, portfolio theory generally suggests ten or more assets should be included. This gives great burden in computation for more complicated types copula functions. We currently cannot find approaches to deal with copulas with ten or more dimensions except for Gaussian and t copula. This is the reason why only these two are utilised. However, even with simple dependence structures like normal and t copula, we can still improve non-normality estimation by fitting univariate empirical cdf of each currency return instead of simple assumption of normal distribution.

The results of Gaussian and t copula portfolio are reported as follows.

Table 5

Jan 2009	Gaussian Copula					
	Foreign Debt Constraints			Foreign Trade Constraints		
	RW	PF	UIP	RW	PF	UIP
USD	40.84%	40.84%		6.53%	6.53%	
EUR	3.31%	3.31%	3.31%	6.45%	6.45%	6.45%
JPY	4.57%	4.57%	4.97%	5.21%	5.21%	5.21%
KRW	41.69%	4.92%	1.18%	57.31%	42.76%	3.63%
SGD	1.18%	1.18%	35.18%	1.02%	1.02%	33.10%
MYR	1.18%	1.18%	1.18%	1.04%	1.04%	1.04%
AUD	1.18%	1.18%	1.18%	1.14%	1.14%	1.14%
CAD	1.18%	1.18%	3.25%	0.67%	0.67%	2.65%
RUB	1.36%	1.18%	39.00%	18.69%	1.11%	37.77%
THB	1.18%	1.18%	6.87%	0.80%	0.80%	6.33%
GBP	1.18%	1.18%	1.18%	0.89%	33.02%	0.89%
CHF	1.18%	1.18%	1.18%	0.22%	0.22%	0.22%
GLD	0.00%	36.95%	1.55%	0.00%	0.00%	1.55%
RW=Random Walk		PF=Perfect foresight		UIP=Uncovered Interest Parity		

Table 6

Jan 2009	T Copula					
	Foreign Debt Constraints			Foreign Trade Constraints		
	RW	PF	UIP	RW	PF	UIP
USD	40.84%	40.84%		6.53%	6.53%	
EUR	3.31%	3.31%	3.31%	6.45%	6.45%	6.45%
JPY	4.57%	4.57%	4.57%	5.21%	5.21%	5.21%
KRW	28.88%	4.93%	1.18%	43.46%	42.76%	3.63%
SGD	1.18%	1.18%	35.55%	1.02%	1.02%	33.14%
MYR	1.18%	1.18%	1.18%	1.04%	1.04%	1.04%
AUD	1.18%	1.18%	1.18%	1.14%	1.14%	1.14%
CAD	1.18%	1.18%	3.62%	0.67%	0.67%	3.09%
RUB	14.17%	1.18%	39.11%	32.54%	1.11%	37.78%
THB	1.18%	1.18%	6.34%	0.80%	0.80%	5.75%
GBP	1.18%	1.18%	1.18%	0.89%	33.02%	0.89%
CHF	1.18%	1.18%	1.18%	0.22%	0.22%	0.22%
GLD	0.00%	36.95%	1.63%	0.00%	0.00%	1.63%
RW=Random Walk		PF=Perfect foresight		UIP=Uncovered Interest Parity		

From what the three tables show, there are 18 different possible portfolios which are resulted by different assumptions. To generate a relative comprehensive suggestion for the currency composition of foreign reserves of China, comparison needs to be made between different situations. Besides three different ways of VCM estimation, there are two other groups of differences, assumption about expected returns and currency weights constraints. We will manifest what the differences reveal in the following analysis.

We begin with common and individual place of three VCM hypotheses. Common characteristic of simple historical and normal copula assumption lies in they both possess the same dependence structure of multi-normal distribution. However, their margins differ in their own univariate distributions. The normal copula assumption allows for empirical distribution functions of each currency instead of total normality. T copula method assumes empirical cdf same as normal copula, but differs with it in sense of dependence structure.

From the reserve allocation result, we can see that normal copula and t copula resemble each other while simple historical deviates with them. Thus, we can tentatively deduce that difference in dependence structure gives relative little effects, while the non-normality assumption of marginal distributions is a more influential factor. The copula estimations are more reliable than simple historical results.

The second difference is the assumptions of expected currency returns. As specified in the methodology section, three assumptions --- random walk, perfect foresight and uncovered interest parity, mean different expected return vectors when optimising. The UIP results do not include a share of USD because it presumes equal returns of all risky currencies, therefore no share of risk free USD. The optimal portfolio is least variance allocation. Thus, equalised returns give most weights of investment on currencies which decrease the whole risk only due to their volatility feature. As a result, RUB and SGD are attractive because they can counter risk. THB is a third best choice. Surprisingly, GLD whose volatility is large is assigned with weight worth noticing. This is because it has small covariance with most currencies. Such allocation of foreign reserves is viable for long run, since UIP is evidenced to be held generally only on long term.

Both random walk and perfect foresight assumptions optimise Sharpe ratio. Without the returns equalising assumption --- UIP, they take the estimated currency returns into consideration. Random walk assumes only interest rates retain in the currency return while perfect foresight assumes the central bank of China has information advantage to predict future FX changes. According to the results under RW, KWR and RUB get big share. It is because of their high interest rate, especially for KWR. It is the most weighted currency except for USD in all four RW categories. When comparing results under assumptions of RW and PF, the most distinctive reserve asset is GLD. Gold itself has relatively low interest return and high volatility, but if its future changes are predictable to some extent by the presuming information advantage of the central bank, it will be a good choice. Allocations under these two assumptions fit the short term investment demand.

The third difference is the foreign finance or trade restrictions on the allocation weights. These constraints are imposed for the purpose of ensuring the basic functions of the foreign reserves. As the results tables reveal, many weights of currencies remain at the levels of the foreign debt or foreign trade restrictions. These currencies are often international currencies such as USD, EUR, JPY, GBP and CHF. To the contrary of conventional thoughts, if it is not for the reason of foreign finance and trade activities, these international currencies are not ideal choice in the consideration of return and risk to China. This phenomenon happens because different currency systems among countries in the portfolio. Countries issuing international currencies use floating exchange rate systems, which cause larger volatility. In terms of minimising risk, investing in pegged currencies seems like a better choice.

5 Conclusion

Since the financial crisis happened in East Asia in the late 1990s, emerging economies have learned the necessity of foreign reserves accumulation to prevent such similar thing. China is one of these countries. As the amount of foreign reserves of China increase sharply, the focus of the question to manage the large amount of foreign reserves gradually turns from the optimal amount of foreign reserves to the optimal currency composition of the reserves. Literatures have shed lights on the topic of optimizing the currency structure of a specific country, and the mean-variance analysis has been proven to be the most effective method. The most comprehensive research in this area by far is the Papaioannou, Portes and Siourounis (2006). It adapts the method of DCC-GARCH into the mean-variance framework. However, an increasing number of researches are criticizing one common assumption of these models --- the multivariate normal standard distribution hypothesis. Therefore we propose a copula modification to handle this problem and based on such analysis we attempt to give some suggestions for the problem of currency composition of foreign reserves for the central bank of China.

Through the comparison of simple historical method of VCM estimation with normal and t copula estimations, we find non-normality exists in the fluctuation of currency returns, and the results by copulas are similar and more reliable. To give a suggestion on the allocation of foreign reserves, UIP assumption analysis represents long term optimal portfolio, with favourite currencies like SGD, RUB, THB and GLD. For short term objective of diversification, random walk and perfect foresight assumptions give two possible approaches. If the central bank of China is just like everyone else in the market and believes in efficient market of foreign exchange, results under RW assumption should be adopted and KWR and RUB are of interest. If the central bank possesses some advantage in prediction of the future trend of the market, results under PF show GLD is a good investment. Another implication of the present paper is when take currencies other than the major international currencies into consideration, these pegged currencies seems more attractive in terms of return and risk motive. However, to achieve the goal of crisis prevention pool, foreign reserves of China

should assign great weight on currencies being frequently used in foreign financing and trade activities.

There are some points needed to be improved in the present paper. First is in the aspect of copula application. It lacks statistical test of the goodness of fit of the copula, and the comparison of copula method and simple historical method are not supported by statistical measures. Few copula dependence structures are used because inability to compute 10 or more dimensional Archimedean copulas. Only unconditional copulas are utilized, which means the time series data are identically and independently distributed. Conditional copula can provide better model to fit data and to provide forecast. These should be attempted in future works. With respect to the reference currency (or numeraire), only a dollar perspective is offered in the present paper. Native currency and currency composite index should be made for comparison in the future. For a larger picture, efforts can be made on modification of the mean-variance framework. Instead of the current single-period framework, multi-period or continuous analysis can be proposed. Another path of research other than mean-variance analysis is to follow the suggestion of Dooley (1986) to use asset-liability perspective to manage of composition.

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