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The Relationship between the Number of Grouped Sample Household and the Gini Coefficient

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Abstract. The literature review indicates that the main problem of calculating the Gini coefficient of Chinese residents' income is the limitation on the data resources. Currently, the group data of incomes of urban and rural residents provided by the Yearbooks is so little that it affects the accurate estimation on the Gini coefficient. According to our research, to increase the group number of income can improve the precision of calculating the Gini coefficient, but the rate of increase in the Gini coefficient shows rapid convergence trend with raise of group number. With empirical analysis, we believe that if we can provide 16 or 20 income sample groups divided by population, we can more accurately measure the overall Gini coefficient of the sample. Therefore, for further study on the income gap, we suggest that the statistics department provide more detailed group data on residents' income.

Keywords: Gini coefficient, group number, decomposition of the Gini Coefficient

1. Introduction

There are various means to measure the income inequality. Among these, there are Theil index, Coefficient of Variation, Kuznets Index, and the Gini Coefficient, etc. However, Lorenz Curve and the Gini Coefficient are most commonly used (Sloman, 2000). The Gini coefficient, however, is the most important index to measure or estimate the income inequality (Sen, 1997; Champernowne and Cowell, 1998). In 1912, Italian statistician, Corrado Gini, published *Variability and Mutability* in which he proposed a method to measure inequality. This method gradually evolved into the well-known Gini Coefficient (Shi Li, 2002).

The ideal statistic source for calculating the Gini Coefficient would be the original household survey data. One can easily and accurately calculate the Gini through certain software[1] based on the formula (1).

$$G = \frac{1}{2n^2 u}\sum_i \sum_j \left| y_i - y_j \right| \quad (1)$$

In formula (1), the n stands for the population (or the family counts); u is the average income; y_i and y_j stand for the income of i family and j family, respectively.

Several statistical yearbooks have been published, such as *China Statistical Yearbook*, *Yearbook of Urban Living and Price Index*, *Yearbook of the Rural Household Survey*[*], and *Statistical Yearbook of Chinese Price Index and Urban Household Survey of Income and Expenditure*. The last two grouped the sampled households in a similar way as the *China Statistical Yearbook.* Although *Yearbook of Urban Living and Price Index* provided more income groups than *China Statistical Yearbook* [†], it started to be officially published only in 2006. However, the publicized income data are not the original ones anymore. The original ones were divided into groups based on income level.

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[*] *Yearbook of the Rural Household Survey* was first published in 1992. However, it was until 2000 that the yearbook started to be published consecutively.
[†] *Yearbook of Urban Living and Price Index* categorized all samples into 20 groups based on their income.

China Statistical Yearbook is the most important data resource to calculate the Chinese Gini ratio, especially when no household survey data was available before 2000. Zongsheng Chen and Yunbo Zhou (Zongsheng Chen, 1999; Yunbo Zhou and Zongsheng Chen, 2002) calculated the Gini coefficient based on this yearbook. However, the income provided in this yearbook has been greatly doubted. Khan and Riskin (2001) commented that the statistical data in the yearbook was too aggregated to prohibit careful analysis of income inequality. Fang, Zhang, and Fan (2002) also believed that the aggregated data ignored the income disparity within each group, and thus they are not accurate enough. In 1996, *China Statistics Yearbook* presented that those with income more than RMB2000 was 38.4% among all the households in the rural area. However, the Yearbook did not divide such households into subgroups anymore. In 2007, the Yearbook presented that those with income more than RMB5000 was 30.94% among all the households in the rural area. Yet after almost 10 years, it still did not provide any more subgroups. In view of these, with the data provided by *China Statistics Yearbook*, we can calculate the income gap among the different groups, while the income gap in the group cannot be calculated, which leads the final result to be underestimated.

Due to the need for large and complex statistical surveys, it is difficult for individuals or non-statistical departments to gain the continuous first-hand data on income of Chinese residents. If we can access the original data source, then we can calculate the Gini coefficient quickly and accurately with Formula (1)[‡]. However, this ideal status is still far-fetched with the current data sources, there are still many technical difficulties needed to be overcome. Although we all know that the current group data of income will lead to underestimation of the Gini coefficient, empirical analysis has not been yet found in the literature. In addition, we know that the Statistical Yearbook can not provide the original income data of the household, but too little group data, such as the current group of five or seven, does not help analyze the income disparity. Therefore, this paper discusses the relationship between the group number of samples of residents' income and calculation of the Gini index. Another problem associated with this problem is how many groups can help estimate accurately the Gini ratio.

The structure of this paper is as follows: the first part poses the problem; the second part will analyze the impact of group data on the calculation of the Gini coefficient; the third part will analyze the impact of group data on the calculation of the Gini coefficient with empirical analysis; the last part makes the conclusion of this paper.

2. The Impact of Grouped Data on the Calculation of the Gini Coefficient

This paper uses decomposition of the Gini coefficient to explain the impact of group data on the calculation of the Gini coefficient. To decompose the Gini coefficient from different sources of income has been well known for everyone, but from different income groups is relatively more difficult. The earliest research in this field is the study of Bhattacharya and Mahalanobis (1967). Pyatt (1976), Mookherjee and Shorrocks (1982), Shorrocks (1984), Lambert and Aronson (1993), Cowell (1995) and Yao (1997) had also made contributions to the decomposition of the Gini coefficient. In brief, the Gini coefficient of all residents can be broke down in the following (See Yao, 1997）：

$$G = G_g^{'} + \sum_i^n P_i I_i G_i + G(f) \quad (2)$$

Here the G_g stands for the Gini coefficient among different groups; G_i stands for the Gini ratio within the group *i;* the proportion of the income of group *i* by the total income is I_i; the proportion of the population of group *i* by the total population is P_i; the value of *G*(*f*) depends on how much the overlap there is among the different groups. If there is no overlap, the *G*(*f*) would be 0 (Mookherjee & Shorrocks,1982; Shorrocks & Wan, 2005). This formula explains why the Gini ratios are underestimated based on the statistical yearbooks. In *China Statistical Yearbook*, the income of city and rural residents were ranked from low to high, therefore, the *G*(*f*) within either the city group or the rural group was 0. However, when using the data from the *China Statistical Yearbook*, there is no way to calculate the difference within each groups, which is to ignore the $\sum_i^n P_i I_i G_i$ within the Formula (2), thus the final results are led to be underestimated.

[‡] Such software includes "DAD" developed by LAVAL, which is able to calculate the index for extreme income inequality. Shi Li (2002) also pointed out that one can use software specializing in calculating the inequality index such as INEQ, or Stata to calculate the Gini ratio. All calculations in this study are done by the Matlab.

If the income of all residents ranks from low to high, then in formula (2), $G(f)$=0, and then the Gini coefficient of income of all residents is as follows:

$$G = G_g^{'} + \sum_{i}^{n} P_i I_i G_i \quad (3)$$

If the income of all residents ranks from low to high, and divided it into average two groups according to the population, then the Gini coefficient of income between the two groups is as follows:

$$G = G_2^{'} \quad (4)$$

If the population is divided into average four groups, the Gini coefficient of income between groups is:

$$G = G_4^{'} = G_2^{'} + \sum_{i=1}^{2} I_i P_i G_i \quad (5)$$

In the formula (5), $G^{'}_2$ is the same with $G^{'}_2$ in the formula (4). The formula (5) is equivalent to firstly put the whole population into two average groups and calculate the Gini coefficient between the two groups, and then to calculate the Gini ratio in the two groups respectively （$\sum_{i=1}^{2} I_i P_i G_i$）.

If the population were divided into eight groups, the Gini coefficient of income among the groups is as follows:

$$G = G_8^{'} = G_2^{'} + \sum_{i=1}^{4} I_i P_i G_i \quad (6)$$

If the population were divided into 2^n (n is positive even number) groups, the Gini ratio of income among the groups is as follows:

$$G = G_{2^n}^{'} = G_2^{'} + \sum_{i=1}^{2^{n-1}} I_i P_i G_i \quad (7)$$

Further, it is not difficult to prove that,

$$G_2^{'} =< \frac{1}{2} 、 \sum_{i=1}^{2} I_i P_i G_i =< \frac{1}{2^2} 、 \sum_{i=1}^{4} I_i P_i G_i - \sum_{i=1}^{2} I_i P_i G_i =< \frac{1}{2^3}$$

Here,

$$\sum_{i=1}^{2^{n-1}} I_i P_i G_i - \sum_{i=1}^{2^{n-2}} I_i P_i G_i =< \frac{1}{2^n} \quad (8)$$

The formula (8) shows that if the average groups are divided by population, the number of groups doubles with the increase in the Gini coefficient by an attenuation of at least a 2 negative exponent one, namely, with the raise in the numbers of the group, the Gini coefficient of the entire population will increase, but the growth shows the convergent trend.

If we set the acceptable calculation error, we can determine the lowest limit on the number of groups, for instance, if there is the sample of 16 average groups, the calculation error of the Gini coefficient must be smaller than $\frac{1}{2^4}$; if 32 average groups, the calculation error must be smaller than $\frac{1}{2^5}$; if 64 average groups, the calculation error must be smaller than $\frac{1}{2^6}$; the calculation error of 128 average groups must be smaller than $\frac{1}{2^7}$.The results are the theoretical limits of these errors, the actual error will be much less than the upper limit of these errors. Here, in the empirical perspective we examine the impact of number of groups on the final outcome of estimation.

3. The Influence of Grouping Number on Calculating the Gini Coefficient

3.1. The Distribution Function of Household Income

In order to study the influence of grouping number on the final calculating results, we study the influence of grouping number by fitting the distribution of China's urban and rural resident income from 2004 to 2006.

Before fitting the resident income distribution, we need to know what type of distribution function can be used to depict the resident income. Steyn (1959) claimed that the income distribution of the same type of people, such as rural residents, could be described soundly by lognormal distribution. Balintfy and Goodman (1973) emphasized that income distribution is generated in a special random process which can be explained by using lognormal distribution. This can also be proved with some empirical results. In its study launched in 2006, the World Bank proved the lognormal distribution of the income, using the data on the resident income in both developed and developing countries in nearly 40 years (Lopez and Servén, 2006). In the study of Japanese resident income from 1887 to 1998, Souma (2000) pointed out that lognormal distribution is the universal structure of resident income distribution. Due to the dual social structure in China, there exists a great gap between urban and rural income. The income distribution of all the residents is in fact more similar to a dumbbell type. Nonetheless, if urban and rural residents' income distributions are observed respectively, it is easy to find that they still obey lognormal distribution

Suppose the resident income x_i (i = 1, 2, ···, n) is an independent and identically distributed random variable, and obeys lognormal distribution with μ and δ as parameters. Its density function is:

$$p(x;\mu,\delta)=\frac{1}{\delta x\sqrt{2\pi}}\exp[-\frac{(\ln x-a)^2}{2\delta^2}] \quad (9)$$

Suppose μ and δ are respectively the mean and the variance of lognormal distribution (see the above formula). According to Hong Xingjian and Li Jinchang's (2006) calculation, for any income variant x, if Lnx～N (μ，δ^2), then the Gini coefficient:

$$G=2\Phi(\frac{\delta}{\sqrt{2}})-1 \quad (10)$$

With equation (10), for any δ, the Gini ratio can be easily calculated with the table of standard normal distribution function. On the contrary, here μ and δ are calculated by using the intra-urban Gini coefficient, the intra-rural Gini coefficient and the urban/rural per capita income provided by the National Bureau of Statistics of China. Since $Y=\exp(\mu+\frac{\delta^2}{2})$ (where Y is the known urban/rural average income), equation (11) can be obtained:

$$\begin{cases}\delta=\sqrt{2}\Phi^{-1}(\frac{G+1}{2}) \\ \mu=\ln(Y)-\frac{\delta^2}{2}\end{cases} \quad (11)$$

The intra-urban Gini coefficients and the intra-rural Gini coefficients from 2004 to 2006 are provided by the Urban Socio-Economic Survey Corps and the Department of Rural & Social Economic Survey of National Bureau of Statistics of China. Through programming with Matlab, after inputting the intra-urban Gini coefficient and the intra-rural Gini coefficient, we can calculate the corresponding δ whose range is accurate to the 4th decimal place. With δ and the known urban-rural per capita income (Y), μ can be calculated using equation (11). Hence, the distribution function of the urban-rural residents' income between 2004 and 2006 can be obtained. Then with Matlab, we can generate the random numbers that obey lognormal distribution with parameters μ and δ. Denoting income of an individual resident, these random numbers can approximately fit the real income data. In our real computations, there are in total 10,000 Matlab-generated random numbers that obey lognormal distribution. According to formula (1), we calculated the urban Gini coefficients and the rural Gini coefficients in all kinds of grouping conditions.

On calculating the national Gini coefficient, we controlled the ratio of random numbers according to the real urban-rural population, and the random numbers was also 10,000. Due to our purpose was not to accurately calculate the national Gini coefficient, but to generate random numbers that obey some distributing rule, we just made use of the random numbers generated by one calculation. In another research of calculating the national Gini coefficient, we used the mean number in 100 times of simulation.

Besides the above methods, we can estimate parameters μ and δ by Expectation-Maximization (EM) algorithm (Deng Ming and Yang Yi, 2004). When calculating μ and δ, we have to know average income of the samples and the Gini coefficient. Moreover, the precondition of using EM algorithm is that we have to know the grouping data of the samples and the grouping data must be enough, or it will affect the final result.

EM algorithm, as an iterative method, is mainly utilized to find the mode of posterior distribution, i.e., maximum likelihood estimates. Iteration consists of two steps: the E-step (Expectation) and the M-step (Maximization). The basic idea of EM algorithm can be manifested as this: With θ as the unknown parameter, we iterate the two steps until □$\theta^{i+1}-\theta^{i}$□is minimized. EM algorithm is simple and stable.

3.2. Results

According to formula (11), using the intra-urban Gini coefficients and the intra-rural Gini coefficients from 2004 to 2006 which are provided by the Urban Socio-Economic Survey Corps and the Department of Rural & Social Economic Survey of National Bureau of Statistics of China, we generated the main parameters of urban/rural residents' income distribution from 2004 to 2006（Table 1）.

Table 1: The main parameters of urban/rural residents' income distribution from 2004 to 2006

Year	Parameters	Rural residents' income	Urban residents' income
2004	μ	7.7538	8.9756,
	δ	0.6797	0.5920
2005	μ	7.8489	9.0886
	δ	0.6914	0.5829
2006	μ	7.948	9.1837
	δ	0.6886	0.6143

Note: The table is from the calculation by the author of the article, based on the *China Yearbook of Rural Household Survey* (2005-2007) and *China Statistical Yearbook of Price and Urban Household Survey* (2005-2007).

According to table 1, utilizing the method of generating random number, we got 10,000 samples of urban/rural residents' income respectively. Then we classified the samples into 2 groups, 4 groups, 8 groups, 16 groups, 20 groups, 40 groups, 80 groups, 200 groups, 1000 groups. Utilizing our own program, we calculated the Gini coefficients of different groups. Table 2 is the result of our calculation.

Table 2. The Gini coefficients of urban/rural residents' income of different number of groups

The number of groups	The Gini coefficients of urban residents' income						The Gini coefficients of rural residents' income					
	2004		2005		2006		2004		2005		2006	
	G	%	*G*	%	*G*	%	*G*	%	*G*	%	*G*	%
2	0.2224	68.45	0.2195	68.74	0.2299	68.3	0.2511	67.88	0.2556	68.27	0.2548	68.07
4	0.2937	90.4	0.289	90.51	0.3036	90.2	0.3328	89.97	0.3379	90.25	0.3372	90.09
8	0.3516	97.14	0.3102	97.15	0.3267	97.06	0.3587	96.97	0.3634	97.06	0.3631	97.01
16	0.3222	99.17	0.3166	99.15	0.3336	99.11	0.3666	99.11	0.3712	99.15	0.371	99.12
20	0.3231	99.45	0.3157	99.44	0.3345	99.38	0.3677	99.41	0.3722	99.41	0.3721	99.41
40	0.3244	99.85	0.3187	99.81	0.3359	99.79	0.3693	99.84	0.3738	99.84	0.3737	99.84
80	0.3248	99.97	0.3191	99.94	0.3365	99.97	0.3697	99.95	0.3742	99.95	0.3741	99.95
200	0.3249	100	0.3192	99.97	0.3366	100	0.3699	100	0.3744	100	0.3743	100
1000	0.3249	100	0.3193	100	0.3366	100	0.3699	100	0.3744	100	0.3743	100

Note: the percentage in each volume represents the rate of different group's Gini coefficient by the Gini coefficient of total 10,000 samples.

The equation (8) shows the relationship between the convergent speed of Gini coefficients and grouping number. We listed the maximum increase of Gini coefficients which was caused by increase of grouping number, but in fact, the real increase of Gini coefficients was smaller. Table 2 shows that as the grouping number increasing from 2004 to 2006, the urban/rural residents' income Gini coefficients increase, too, with a faster convergent speed. When we grouped the samples into 2 groups, the rate of group's Gini coefficient by the Gini coefficient of total 10,000 samples is more than 2/3. When we grouped the samples into 16 groups, the rate becomes to 99%. And the rate surpasses 99% when we grouped the samples into 20 groups. Suppose the error standard is 1%, the Gini coefficient of 16 groups and 20 groups can basically reflect urban residents' income Gini coefficients or rural residents' income Gini coefficients. We discussed the relationship between urban/rural residents' income Gini coefficients and grouping number of the samples. Furthermore, what's the relationship between the national Gini coefficient and grouping number?

Table3. The national Gini coefficient and the Gini coefficient of Sichuan urban inhabitants

The number of groups	The nation-wide Gini coefficient						The Gini ratio of Sichuan urban inhabitants	
	2004		2005		2006		December, 2008	
	G	%	*G*	%	*G*	%	*G*	%
2	0.3105	68.83	0.3124	69.05	0.3148	68.63	0.225	65.77
4	0.4083	90.51	0.4100	90.62	0.4148	90.43	0.3029	88.54
8	0.4383	97.16	0.4399	97.23	0.4457	97.16	0.3290	96.17
16	0.4473	99.16	0.4488	99.20	0.4549	99.17	0.3378	98.74
20	0.4486	99.45	0.4500	99.47	0.4562	99.45	0.3392	99.15
40	0.4504	99.84	0.4517	99.91	0.4580	99.84	0.3412	99.74
80	0.4509	99.96	0.4522	99.96	0.4585	99.96	0.3418	99.91
200	0.4511	100	0.4524	100	0.4587	100	0.3421	100
1000	0.4511	100	0.4524	100	0.4587	100	0.3421	100

Note: the percentage in each volume represents the rate of every group's Gini coefficient by the Gini coefficient of total 10,000 samples.

The method of calculating the national Gini coefficient is different from the one of urban/rural residents' income Gini coefficients. Take the calculation of the national Gini coefficient in 2006 for example, as the percentage of urban residents is 43.9%, according to the parameters of urban residents' income distribution function in table 1, we generated 4,390 random numbers. In the same way, we generated 5,610 random numbers according to the parameters of rural residents' income distribution function in table 1. With the combination of the 2 parts of random numbers, we got the samples of national residents' income, and we calculated the Gini coefficients of different groups according to these samples. Through table 3, we found that the relationship between the national Gini coefficient and grouping number is the same as the relationship between urban/rural residents' income Gini coefficients and grouping number. If we grouped both urban and rural residents into 16 groups or 20 groups, the error of the calculated national Gini coefficient is less than 1%.

However, the above analysis is based on random numbers which obey special parameters. Whether the real statistics support our analysis? We examined the relationship between Gini coefficient and grouping number with the urban residents' statistic samples of Sichuan province in December, 2008. The samples surpassed 10,000 rural families. The result showed that the urban residents' income Gini coefficient of Sichuan province was 0.3421 in December, 2008. After arranging the urban residents' income of Sichuan province from low to high, we grouped the samples into 2 groups, 4 groups, 8 groups, 16 groups, 20 groups, 40 groups, 80 groups, 200 groups and 1000 groups to calculate the Gini coefficient of different groups respectively. The result was the same as the calculation based on random numbers which obey special parameters. When we grouped the samples into 2 groups, the rate of group's Gini coefficient by the Gini coefficient of total 10,000 samples is more than 2/3. When we grouped the samples into 16 groups, the rate becomes to 99%. And the rate surpasses 99% when we grouped the samples into 20 groups.

In conclusion, if the Year Book could provide statistic data of residents' income by 16 groups or 20 groups with the same population, it won't be difficult to calculate the Gini coefficient of the relevant samples accurately.

4. Conclusion

The data source at present is responsible for the puzzles on China's national income Gini coefficient. Furthermore, we cannot estimate accurately because of the lack of urban/rural grouping data which were provided by the yearbook. The research shows that increase of grouping number can improve the calculation precision of Gini coefficient, but the increase extent will be convergent very fast as the increasing of grouping number. Through empirical analysis, we believe that we can calculate the Gini coefficient of the relevant samples accurately if the income data of 16 groups or 20 groups with the same population can be provided. So we suggest that the statistic departments provide more details about grouping data. At the same time, we also suggest to improve the calculation measures in order to overcome the limitation of data source. In this article, through the method of back-calculation of urban/rural income distribution function, we try to put forward a new method to calculate the national Gini coefficient.

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