Precautionary Reserves of Emerging Economies: a Behavioural Approach under Loss Aversion

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Abstract

In this paper, we extend the optimising model of Valencia (2010) to incorporate the idea of loss aversion into the evaluation of reserves accumulation held by emerging economies as a means of precautionary savings against possible economic adversaries including financial crisis. The theoretical underpinning and empirical calibration in this paper suggests that, compared to the outcome offered by traditional models, the interpretation of reserves accumulation is more realistic under loss aversion.

Key words: International reserves, precautionary motive, loss aversion.

1. Introduction

Essentially, what the optimal level of international reserves is has become one of the most vital issues to be necessarily faced by the emerging countries, particularly under the recent circumstance where countries with the export-oriented economy hoard a huge amount of reserves as a self-insurance to cushion the current financial crisis. Needless to say, so far, there have been a mass of theoretical and empirical studies involving in this issue (e.g. Bahmani-Oskooee and Brown (2002) for a review).

The earlier theories of reserves accumulation chiefly concentrate on the current account. Representatively, Heller (1966) proposes that the optimal level of reserves is triggered by the equivalently marginal amount of both benefit and cost, which is analogous to the traditional cost-benefit analysis in microeconomics. Frenkel and Jovanovic (1981) develop a stochastic inventory control model in which reserves serve as a buffer stock to cushion current account deficit. Based on Heller's approach, Ben-Bassat and Gottlieb (1992) generate a precautionary model in which reserves can be used as precautionary purpose for a borrowing country challenged by country risk and the cost of default, based on the fact that central bank can assist its country to alleviate the negative outcome triggered by economic adversaries by virtue of holding international reserves.

In terms of the studies of optimal reserves in recent years, they have two main features: (1) precautionary motive has become a mainstream for the interpretation of reserves accumulation (Aizenman and Lee (2007) – a typical study); (2) in parallel with the trend in macroeconomic theory, the welfare (utility) of the representative agent has been referred to as the standard to maximize when involving in the issue of the evaluation of optimal reserves. Studies on reserves accumulation with these two features have been well documented (e.g.

Caballero and Panageas (2008), Barnichon (2008), Jeanne and Ranciere (2008), and Valencia (2010)).

On the other hand, standard economics under Expected Utility Theory has been systematically violated by so called 'anomalies' based on an army of empirical studies. However, as an alternative to standard economics, behavioural economics has been boosting the explanatory power of economics based on more realistic psychological foundations, the core of which is loss aversion under 'Prospect Theory' pioneered by Kahneman and Tversky (1979). Subsequently, a considerable number of literature have been documented not only on the development of loss aversion itself (e.g. Zank, 2007), but also on its extensive application in both economics and finance (e.g. Camerer, 2000). Undoubtedly, behavioural economics has demonstrated the bounded rationality of individuals when making decisions, and therefore been effectively in response to the anomalies faced by the traditional paradigm.

However, the existing studies on reserves accumulation are still based on the assumption of standard economics (i.e. Expected Utility Theory), in order that their conclusions are not completely in line with the level of reserves observed. In this paper, we extend the optimizing model of Valencia (2010) – the latest study of precautionary reserves – by way of incorporating loss aversion into the evaluation of reserves accumulation held by emerging economies as a means of precautionary savings against possible economic adversaries including financial crisis. The theoretical underpinning and empirical calibration in this paper suggests that, compared to the outcome offered by traditional models, the interpretation of reserves accumulation is more reasonable under loss aversion.

This paper is organized as follows: section two reviews loss aversion under Prospect theory and its comprehensive application; section three introduces the model of Valencia (2010) and generates its extension based on loss aversion and calibrated results; section four shows conclusion.

2. Loss Aversion under Prospect Theory

2.1 Loss Aversion

Loss aversion is the central feature of Prospect Theory (PT) – the core of behavioural economics – pioneered by Kahneman and Tversky (1979), which indicates that the value function (i.e. $U = U(x)$) is defined on changes in wealth (i.e. x) but not on final asset position based on traditional economics, and that the agents always refer to outcomes as gains (i.e. $x > 0$) or losses (i.e. $x < 0$) relative to a certain reference point (i.e. $U(0) = 0$) and are more sensitive to losses than to completely commensurate gains (i.e. more weights are assigned to losses than equally sized gains) (Kahneman and Tversky, 1979). The other two features are the probability weighting function and the concavity (convexity) of the value function over gains (losses) (i.e. U is concave for $x > 0$ and convex for $x < 0$, i.e. it is S-shaped).

Tversky and Kahneman (1992) extend their original prospect theory in 1979, by virtue of the rank-dependent utility theory by Quiggin (1982), to generate the cumulative prospect theory (CPT). There are three functions to illustrate loss aversion under CPT: the value (utility) function U , the weighting function for gain probability w^{+} , and that for loss probability w^{-} . Specifically, they propose a piecewise power function as the value function denoted by $U(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\beta} & if\ x < 0 \end{cases}$ and two weighting functions expressed by

$w^+ = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$ and $w^- = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$, respectively. By way of experimental data, they evaluate $\alpha = \beta = 0.88$ in line with diminishing sensitivity, $\lambda = 2.25$ implying the degree of loss aversion, $\gamma = 0.61$, and $\delta = 0.69$. Al-Nowaihi *et al.* (2008) generate a formal proof for the power function and two weighting functions proposed in Tversky and Kahneman (1992) and conclude that $\lambda > 1$, $\alpha = \beta$, and $w^+ = w^-$ (i.e. $\gamma = \delta$). Barberis and Huang (2008) suggest that $\delta \in (0.28,1)$ is required in order to ensure the weighting function w is strictly increasing during $p \in (0,1)$ under $\gamma = \delta$.

Subsequently, some relevant studies propose a variety of definitions of loss aversion based on the utility function. For instance, Kobberling and Wakker (2005) introduce an index of loss aversion. Other definitions can be found in Neilson (2002) and Bowman *et al.* (1999). Maggi (2004) summarises those definitions above and imposes some parameters' restrictions on typical S-shaped utility functions in order that they can display loss aversion.

2.2 Applications of Loss aversion

The philosophy of loss aversion has been well incorporated into both economics and finance. Some typical evidence of loss aversion in economics are asymmetric in demand elasticity after price increasing and decreasing (Hardie *et al.* 1993), downward-sloping labour supply based on the fact that cab drivers in New York City quit early after achieving a daily income target (Camerer *et al.* 1997), and purchase strategies of hog farmers (Pennings and Smidts, 2003). Besides, loss aversion can explicate both the status quo bias (i.e. an overstated preference for the status quo) labelled by Samuelson and Zeckhauser (1988) and endowment effects (i.e. the owners always overestimate its value than potential buyers when facing an economically equivalent good) experimented by Kahneman *et al.* (1991).

In addition, behavioural finance has an effective interpretation to anomalies that traditional finance cannot explain. One major anomaly is so called disposition effect pioneered by Shefrin and Statman (1985), which implies that investors have a tendency both to sell stocks that have obtained value (winners) and to hold onto stocks that have lost value (losers) based on their purchase price (i.e. a reference point). Odean (1998) concludes that the significant preferences of investors can be confirmed for selling winner soon and holding losers long.

Another main anomaly is so called the equity premium – the large gap between risky assets (i.e. stocks) and riskless assets (i.e. T-bills). Benartzi and Thaler (1995) firstly expound the equity premium employing loss aversion based on the aggregate stock market, and conclude that investors charge a high return on average to eliminate the discomfort feeling caused by the high volatility of stock return. Barberis, Huang, and Santos (2001) proposed a model of asset prices by considering investors not only getting utility from consumption but also from volatilities in the value of their financial wealth. Based on investors with loss aversion over these volatilities, they finally conclude that their model can facilitate the interpretation of the high return on average, excessive volatilities, and the ability of prediction of stock returns. Barberis and Huang (2009) overcome the limitations of Barberis, Huang, and Santos (2001), and generate an inter-temporal preference specification to demonstrate its tractability in both portfolio choice and equilibrium settings.

It is worth noting that two studies have simply applied loss aversion to the illustration of reserves accumulation so far. Aizenman (1998) employs the disappointment aversion of Gul (1991) – similar to loss aversion – (i.e. asymmetric aversion to gains versus losses) to investigate buffer stock and precautionary savings to show a stabilization fund is rather larger under disappointment aversion (DA) than that under expected utility. The core of model is when the utility function is based on the agent with DA preference facing uncertainty of incomes ($Y+\varepsilon$ or $Y-\varepsilon$) with an equal probability within two periods, more weights are put into losses (i.e. $Y-\varepsilon$) than into gains (i.e. $Y+\varepsilon$) relative to the reference point (i.e. Y). Likewise, Aizenman and Marison (2003) incorporates the idea of loss aversion into the inter-temporal consumption model for budget constraints of two periods, in order to expound large reserve holdings in a number of Asian emerging markets. They assign the extra weights θ to so called 'the bad state of nature' when the agents face a productivity shock in the second period with an equal possibility, and they also indicate that loss aversion ratio is $\frac{1+\theta}{1-\theta}$. They find that the increase in the degree of loss aversion and/or the volatility shocks will boost reserves holdings.

2.3 Summary

In brief, considerable evidence implies that loss aversion has been a ubiquitous phenomenon in economics and finance and it has effectively expounded the puzzles that standard paradigm do not. However, it is not adequately explored for the great potential of loss aversion on the issue of reserves accumulation, in spite of the two relevant studies (i.e. Aizenman (1998) and Aizenman and Marison (2003)). Consequently, it is necessary to further incorporate the idea of loss aversion into interpreting reserves accumulation based on the latest study – Valencia (2010), for the illustration of precautionary reserves in emerging countries using loss aversion, which can be shown in the next section.

3. The Model

3.1 The Model of Valencia (2010)

Valencia (2010) modified a standard precautionary savings model offered by Carroll (2004) to illustrate the precautionary reserves of Bolivia, whose economy is based on commodity export with little reliance on foreign capital inflows, and therefore is more subject to volatilities in export revenues than to sudden interruptions of capital inflows. The core of precautionary savings theory is that households accumulate the amount of extra savings to respond to uncertainty of their future income.

The assumption of his model is that households consume only tradable goods, facing an exogenously given interest rate and income process, and make consumption decisions in order to maximise the expected present discounted value (PDV) of the utility derived from consumption,

$$Max_{\{C_t\}_0^\infty} E_t \sum_t^\infty \beta^t u(C_t) \qquad (1)$$

where β is the discount factor, $u(\bullet)$ is the utility function with CRRA type (i.e. $u(\bullet) = \bullet^{1-\rho} / (1-\rho)$), and E is the expectations operator.

At the beginning of the period, consumers have net foreign assets X_t; after their consumption decisions C_t, the remainder $(X_t - C_t)$ invested in a security with risk-free rate R. At the end of the period, X_{t+1} is the amount when income is realised, which determines over how much assets the consumer holds in period t+1. Such a process can be denoted by the equation

$$X_{t+1} = R(X_t - C_t) + \tau_{t+1} Y_{t+1} + A_t \qquad (2)$$

where Y is the level of permanent income, τ indicates transitory shocks to income. A is all other non-export net current receipts. The consumer is under a borrowing constraint that consumption will be as high as the current level of net foreign assets.

$$C_t \leq X_t \qquad (3)$$

For simplicity, the author assumes that $Y_{t+1} = Y_t$, which means that transitory shocks is the only source of income variation. The equations above are divided by the level of permanent income Y_t for the normalisation. Corresponding small letters denote the normalised version of capital letters after the normalisation. The optimisation problem can be obtained by solving the Bellman's equation as discussed below.

$$V(x_t) = Max_{\{C_t\}_0^\infty} \{u(c_t) + \beta E_t V(x_{t+1})\} \qquad (4)$$

subject to

$$x_{t+1} = R(x_t - c_t) + \tau_{t+1} + a \qquad (5)$$

$$c_t \leq x_t \qquad (6).$$

The first order condition of the problem is expressed by

$$u^{'}(c_t) = R\beta E_t V^{'}(x_{t+1}) \qquad (7),$$

which means that the marginal utility of consumption should be the same as the expected PDV of the marginal utility of holding net foreign assets. Using the envelope theorem, equation (7) can be denoted by

$$u^{'}(c_t) = R\beta E_t u^{'}(c_{t+1}) \qquad (8),$$

which sets the marginal utility of current consumption equal to the marginal utility of future consumption. It is a necessary condition for the degree of impatience expressed by $R\beta < 1$, which avoids the consumer's wealth growing infinitely (this condition comes from Carroll 2004).

The term τ consists of two components assumed to be independent of each other for simplicity: (1) terms of trade shocks ς; (2) transitory shocks to export volumes γ. Thus, $\tau = \varsigma\gamma$. Here the assumption is $\varsigma_{t+1} = 1$ for all t, which indicates that the only source of uncertainty is volume shocks. For such shocks, the assumption is that there exists a dummy variable such that γ can be expressed by $\gamma = \begin{cases} 1 & p = 1-\theta \\ 0 & p = \theta \end{cases}$, which meaning the economy would suffer from a sudden depletion of resources with the probability of θ.

On the one hand, the solution for the economy with exhausted resources is given by

$$c_t^0 = \underbrace{(1 - R^{-1}(R\beta)^{1/\rho})}_{k} x_t \qquad (9).$$

This equation indicates that the amount of consumption under the economy with exhausted resources is a constant fraction of wealth in every period (because of γ can only take two values, 1 or 0, there is a superscript to the relevant variables that takes the value of 1, i.e. resources available) and 0, i.e. resources depleted).

On the other hand, the solution for the economy with resources available is based on equation (8). By way of combining the equation $u(\bullet) = \bullet^{1-\rho} / (1-\rho)$ and equation (8), the solution can be expressed by

$$1 = R\beta E_t (c_{t+1} / c_t)^{-\rho} \qquad (10).$$

Because of the existence of γ, the expected value in equation (10) can be denoted by

$$1 = R\beta\left[(1-\theta)(c_{t+1}^{1}/c_{t}^{1})^{-\rho} + \theta(c_{t+1}^{0}/c_{t}^{1})^{-\rho}\right] \qquad (11).$$

The equation (10) can be transformed by multiplying both sides $(c_{t+1}^{1}/c_{t}^{1})^{\rho}$ into

$$(c_{t+1}^{1}/c_{t}^{1})^{\rho} = R\beta\left[(1-\theta) + \theta(c_{t+1}^{1}/c_{t}^{0})^{\rho}\right] \qquad (12).$$

From equation (12), it can be seen that, if $\theta = 0$, consumption growth corresponds to the growth under perfect foresight (i.e. $(R\beta)^{1/\rho}$) in equation (10). The concept ‘‘perfect foresight’’ – the benchmark solution – derived from Carroll (2004), which means there is no uncertainty for the income process. For $\theta > 0$, because of $c_{t+1}^{1} > c_{t}^{0}$ (i.e. consumption under resources available is larger than when resources are depleted), the expression (c_{t+1}^{1}/c_{t}^{0}) is greater than one. Thus, consumption growth can be boosted by the presence of uncertainty compared with that under perfect foresight. Based on the fact that the PDV of consumption is equal to the PDV of income (i.e. for the given x_{t+1}), higher consumption growth implies a lower initial level of consumption, which suggests that the introduction of a risk of depleting resources can induce a precautionary increase in savings.

As to the utility function of CRRA type, it assumes that the coefficient of relative risk aversion is set to 1, $\rho = 1$ (i.e. consider the special case of logarithmic utility) for the purpose of simplicity. Taking the equation (12) as the starting point, the optimal level of net foreign assets is given by

$$x = \frac{R}{-R + \left(\frac{(R\beta)^{-1} - 1}{\theta} + 1\right)(R - R\beta) + 1} + 1 \qquad (13)$$

The detailed process can be found in Valencia (2010). Obviously, an increase in θ can give rise to the decrease in the denominator of equation (13), and therefore an increase in the optimal level of reserves (because $R\beta < 1, (R\beta)^{-1} > 1$). By way of calibrating equation (13) (i.e. $\beta = 0.94$ and $R = 1 + r = 1 + 0.02 = 1.02$), the relationship between reserves (x) and the risk (θ) can be plotted in the figure below.

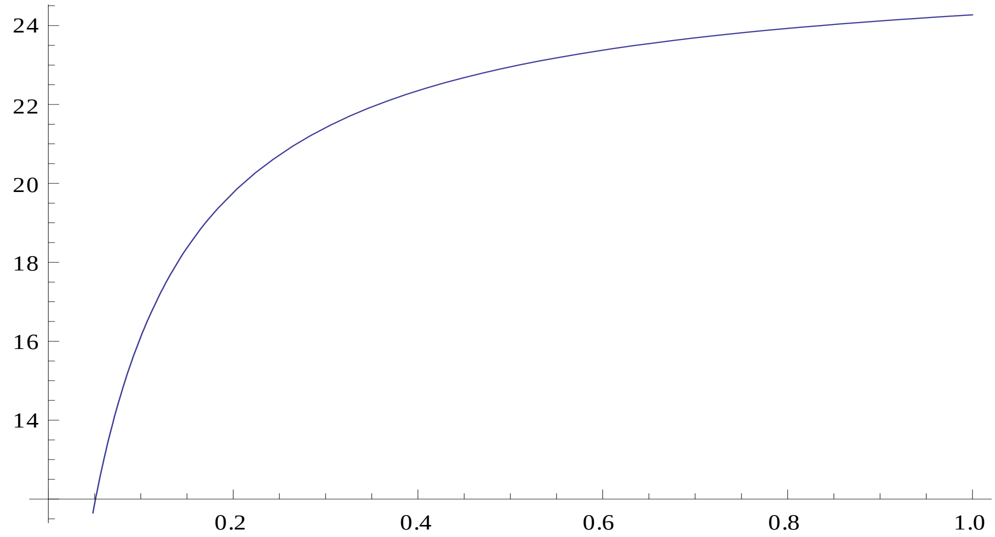

Figure (1)

According to Figure (1), the horizontal axis denotes risk, and the vertical axis denotes the optimal level of reserves. It is can be seen that there is a positive relationship between the optimal level of reserves and the risk of resources depletion, with the larger marginal impact at initially low levels of risk.

3.2 The extension of Valencia (2010)

Based on Valencia (2010), now it is the time to incorporate the idea of loss aversion into interpreting reserves accumulation. Gomes (2005) employed the CRRA utility function as a basis of establishing his behavioural model, denoted by

$$V^0 \equiv \begin{cases} V_G \equiv \dfrac{(W-\Gamma)^{1-\gamma}}{1-\gamma}, & W \geq \Gamma \\ \lambda V_L \equiv -\lambda \dfrac{(\Gamma - W)^{1-\gamma}}{1-\gamma}, & W \leq \Gamma \end{cases} \qquad (14)$$

where Γ is the reference point of the investors, and $\lambda > 1$, denoting the degree of loss aversion. Similarly, the utility function used in our model is implied by

$$u(c) = \begin{cases} \dfrac{(c-c^*)^{1-\rho}}{1-\rho} & c \geq c^* \\ -\lambda \dfrac{(c^*-c)^{1-\rho}}{1-\rho} & c < c^* \end{cases} \qquad (15)$$

where c^* is the reference point (e.g. a certain level of consumption), and $\lambda > 1$ indicating the degree of loss aversion. In terms of Valencia (2010), the only source of uncertainty is volume shocks, as expressed by a dummy variable $\gamma = \begin{cases} 1 & p = 1-\theta \\ 0 & p = \theta \end{cases}$ (θ is the probability of being in the state of resources depletion), a superscript is attached to the relevant variables that takes the value of 1 (i.e. resources available) or 0 (i.e. resources depleted).

The start point of our model is the core equation of Carroll (2004) denoted by

$$u^{'}(c_t) = R\beta E_t u^{'}(c_{t+1}) \qquad (16)$$

which implies that the marginal utility of consumption in the current period should be the same as the expected present discounted value (PDV) of the marginal utility of consumption in the future period. Given the existence of the uncertainty, equation (16) becomes

$$u^{'}(c_t^1) = R\beta E_t u^{'}(c_{t+1}^{\bullet}) \qquad (17),$$

where $c_{t+1}^{\bullet}$ indicates the two possibilities in the future: c_{t+1}^1 (i.e. consumption under resources available) and c_{t+1}^0 (i.e. consumption under resources depleted). Novemsky and Kahneman (2005) find that, ‘‘in its simplest form, loss aversion is applied to all negative departures from the status quo’’. In consequence, c^* is set to be the status quo here (i.e. consumption under resources available) and c_{t+1}^0 is the negative departure from such a status quo, and therefore its utility function will take the second line in (15) (i.e. the utility function when $c < c^*$), which lies in the fundamental difference between behavioural economics and the standard economics.

Thus, equation (17) can be shown to be:

$$u^{'}(c_t^1) = R\beta[\theta \cdot u^{'}(c_{t+1}^0) + (1-\theta) \cdot u^{'}(c_{t+1}^1)] \qquad (18)$$

Combining equations (15) and (18), we obtain

$$(c_t^1 - c^*)^{-\rho} = R\beta[\theta \cdot \lambda \cdot (c^* - c_{t+1}^0)^{-\rho} + (1-\theta) \cdot (c_{t+1}^1 - c^*)^{-\rho}] \qquad (19)$$

So that:

$$1 = R\beta\left[\theta \cdot \lambda \cdot \left(\frac{c^* - c_{t+1}^0}{c_t^1 - c^*}\right)^{-\rho} + (1-\theta) \cdot \left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{-\rho}\right] \qquad (20)$$

Finally, we can derive a formula for illustrating the relationship between the optimal level of net foreign assets (x) and the probability (θ) of when resources are depleted, denoted by

$$x = \frac{-R\left(\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)^{1/\rho} + 1\right)c^{*} + R}{-R + \left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)^{1/\rho}((R\beta)^{1/\rho} - R) + 1} + 1 \qquad (21)$$

The detailed process is shown in Appendix.

Comparing equation (21) to that in Valencia (2010) which is

$$x = \frac{R}{-R + \left(\frac{(R\beta)^{-1} - 1}{\theta} + 1\right)^{1/\rho}(R - (R\beta)^{1/\rho}) + 1} + 1 \qquad (22)$$

We find that: (1) If $c^{*} = 0$ (i.e. the analysis under standard economics – taking the same form of utility function regardless of the existence of reference point or the effect of status quo), the numerators of the two equation will be same;

(2) One term of the numerator in equation (21) – $\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)^{1/\rho}$ will be equal to the corresponding term in equation (22) $-\left(\frac{(R\beta)^{-1} - 1}{\theta} + 1\right)^{1/\rho}$, if $\lambda = 1$ (i.e. no existence of loss aversion below the reference point).

(3) As to the term $((R\beta)^{1/\rho} - R)$, the opposite signs in (21) and (22) are derived from the different signs of c_{t+1}^{0} in (21) and (22).

The differences based on first two points above do depict the core of Prospect Theory – the existence of reference point and the degree of loss aversion, because of which, our final outcome is distinct from that in Valencia (2010).

With regard to equation (21), when $\rho = 1$ (i.e. the special case of logarithmic utility), it can be shown that

$$x = \frac{-R\left(\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right) + 1\right)c^* + R}{-R + \left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)(R\beta - R) + 1} + 1 \qquad (23)$$

Under (23), taking values of plausible parameters ($R = 1 + r = 1 + 0.02$; $\beta = 0.94$; $\lambda = 2.25$; $c^* = 2$), the relationship between reserves (x) and the risk (θ) can be plotted in the figure (2). The horizontal axis indicates risk, and the vertical axis indicates the optimal level of reserves.

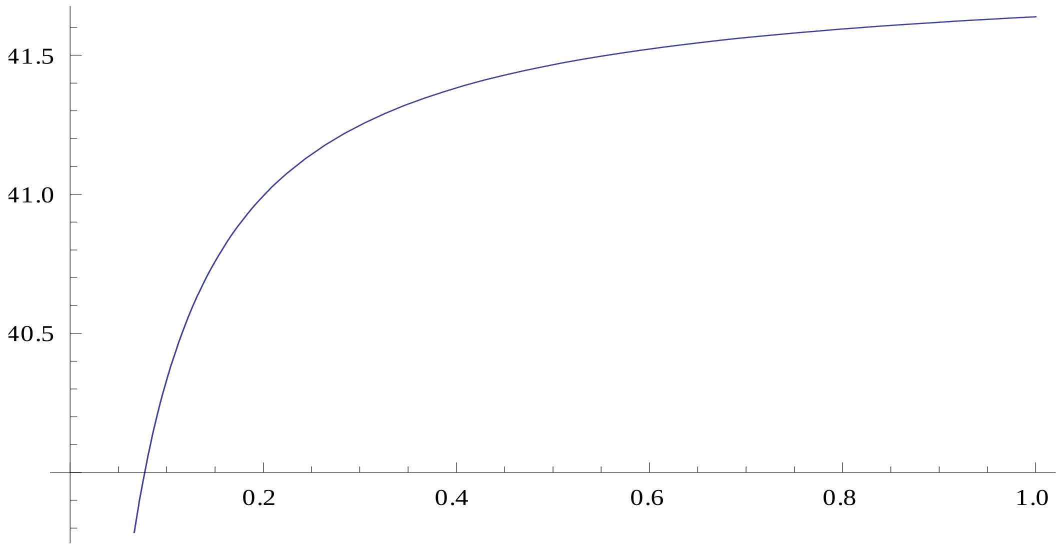

Figure (2)

The corresponding relationship under (22) is revisited below, in terms of Valencia (2010).

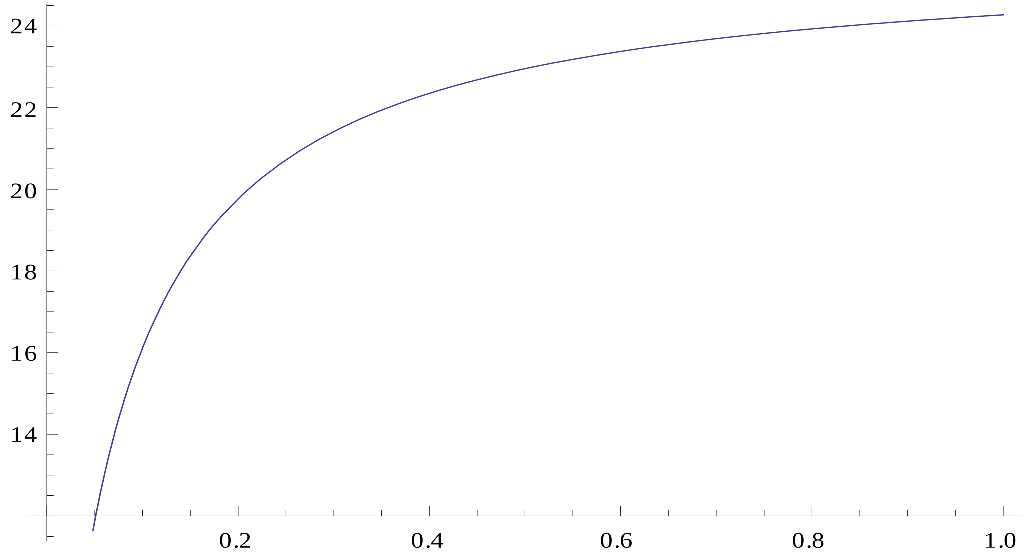

Figure (3)

Despite the seeming similarity of these two schedules, the values of vertical axis based on these two figures are quite different in the general shape. Note that the horizontal axis indicates the amount of net foreign assets (x) which has been normalised. Then, recall that, the equation in capital letters (i.e. $X_{t+1} = R(X_t - C_t) + \tau_{t+1} Y_{t+1} + A_t$) has been divided by the level of permanent income Y_t and shown in small letters (i.e. $x_{t+1} = R(x_t - c_t) + \tau_{t+1} + a$). As a result, according to the two figures, the values of horizontal axis indicate the increasing rates of external assets in relation to the increase in the probability of resource depletion. It can then be seen that, comparing Figure (3), Figure (2) shows that both the increasing rates and the increasing amount of foreign assets around the probability of 10 percent is much larger when considering the idea of loss aversion than those based on the assumption of traditional economics.

4. Conclusion

In this paper, we generate a behavioural approach to interpret the precautionary motive of reserves accumulation by way of incorporating the idea of loss aversion into the optimising model of Valencia (2010). The calibrated result of our model indicates that precautionary reserves held by emerging countries are much larger when considering the idea of loss aversion than those based on the assumption of standard economics.

5. Appendix

The starting point is the equations in Valencia (2010), given by

$$u^{'}(c_t^1) = R\beta E_t u^{'}(c_{t+1}^{\bullet})$$

$$u^{'}(c_t^1) = R\beta[\theta \cdot u^{'}(c_{t+1}^0) + (1-\theta) \cdot u^{'}(c_{t+1}^1)]$$

When considering loss aversion, the equations are shown below:

$$(c_t^1 - c^*)^{-\rho} = R\beta[\theta \cdot \lambda \cdot (c^* - c_{t+1}^0)^{-\rho} + (1-\theta) \cdot (c_{t+1}^1 - c^*)^{-\rho}]$$

$$1 = R\beta\left[\theta \cdot \lambda \cdot \left(\frac{c^* - c_{t+1}^0}{c_t^1 - c^*}\right)^{-\rho} + (1-\theta) \cdot \left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{-\rho}\right]$$

$$1 = R\beta \cdot \left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{-\rho}\left[\theta \cdot \lambda \cdot \left(\frac{c^* - c_{t+1}^0}{c_t^1 - c^*}\right)^{-\rho}\left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{\rho} + (1-\theta)\right]$$

$$1 = R\beta \cdot \left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{-\rho}\left[\theta \cdot \lambda \cdot \left(\frac{c_t^1 - c^*}{c^* - c_{t+1}^0}\right)^{\rho}\left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{\rho} + (1-\theta)\right]$$

$$1 = R\beta \cdot \left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{-\rho}\left[\theta \cdot \lambda \cdot \left(\frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0}\right)^{\rho} + (1-\theta)\right]$$

$$\left(\frac{c_{t+1}^1 - c^*}{c_t^1 - c^*}\right)^{\rho} = R\beta\left[\theta \cdot \lambda \cdot \left(\frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0}\right)^{\rho} + (1-\theta)\right]$$

Setting $c_{t+1}^1 = c_t^1 = \hat{c}$

$$1 = R\beta\left[\theta \cdot \lambda \cdot \left(\frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0}\right)^{\rho} + (1-\theta)\right]$$

$$(R\beta)^{-1} = \theta \cdot \lambda \cdot \left(\frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0} \right)^{\rho} + 1 - \theta$$

$$\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta} \right) = \left(\frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0} \right)^{\rho}$$

$$\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta} \right)^{1/\rho} = \frac{c_{t+1}^1 - c^*}{c^* - c_{t+1}^0} = \frac{c^* - c_{t+1}^1}{c_{t+1}^0 - c^*}$$

Setting $\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta} \right)^{1/\rho} = \mathrm{A}$

$$\mathrm{A} = \frac{c^* - c_{t+1}^1}{c_{t+1}^0 - c^*} = \frac{c^* - \hat{c}}{kR(x_t^1 - \hat{c}) - c^*} \quad (c_{t+1}^0 = kx_{t+1}^0 \text{ and } x_{t+1}^0 = R(x_t^1 - c_t^1))$$

$$\mathrm{A}\left[kR(x_t^1 - \hat{c}) - c^* \right] = c^* - \hat{c}$$

$$\mathrm{A}kRx_t^1 - \mathrm{A}kR\hat{c} - \mathrm{A}c^* = c^* - \hat{c}$$

$$\mathrm{A}kRx_t^1 - (\mathrm{A}+1)c^* = \mathrm{A}kR\hat{c} - \hat{c}$$

$$\mathrm{A}kRx_t^1 - (\mathrm{A}+1)c^* = (\mathrm{A}kR - 1)\hat{c}$$

Setting $x_{t+1}^1 = x_t^1 = x$

Because of $x = R(x - \hat{c}) + 1$, and therefore $R^{-1} - x(R^{-1} - 1) = \hat{c}$

$$\mathrm{A}kRx - (\mathrm{A}+1)c^* = (\mathrm{A}kR - 1)(R^{-1} - x(R^{-1} - 1))$$

$$\mathrm{A}kRx - (\mathrm{A}+1)c^* = (\mathrm{A}kR - 1)(R^{-1} - xR^{-1} + x)$$

$$\mathrm{A}kRx - (\mathrm{A}+1)c^* = \mathrm{A}kR(R^{-1} - xR^{-1} + x) - (R^{-1} - xR^{-1} + x)$$

$$\mathrm{A}kRx-(\mathrm{A}+1)c^{*}=\mathrm{A}k-\mathrm{A}kx+\mathrm{A}kRx-R^{-1}+xR^{-1}-x$$

$$-(\mathrm{A}+1)c^{*}=\mathrm{A}k-\mathrm{A}kx-R^{-1}+xR^{-1}-x$$

$$-R(\mathrm{A}+1)c^{*}=\mathrm{A}kR-\mathrm{A}kRx-1+x-xR$$

$$1-R(\mathrm{A}+1)c^{*}-\mathrm{A}kR=x-xR-\mathrm{A}kRx$$

$$1-R(\mathrm{A}+1)c^{*}-\mathrm{A}kR=x(1-R-\mathrm{A}kR)$$

$$x=\frac{1-R(\mathrm{A}+1)c^{*}-\mathrm{A}kR}{1-R-\mathrm{A}kR}$$

Because of $k=1-R^{-1}(R\beta)^{1/\rho}$

$$x=\frac{1-R(\mathrm{A}+1)c^{*}-\mathrm{A}(1-R^{-1}(R\beta)^{1/\rho})R}{1-R-\mathrm{A}(1-R^{-1}(R\beta)^{1/\rho})R}$$

$$x=\frac{1-R(\mathrm{A}+1)c^{*}-\mathrm{A}(R-(R\beta)^{1/\rho})}{1-R-\mathrm{A}(R-(R\beta)^{1/\rho})}$$

$$x=\frac{-R(\mathrm{A}+1)c^{*}-\mathrm{A}(R-(R\beta)^{1/\rho})+1}{-R-\mathrm{A}(R-(R\beta)^{1/\rho})+1}$$

$$x=\frac{-R(\mathrm{A}+1)c^{*}+R}{-R-\mathrm{A}(R-(R\beta)^{1/\rho})+1}+1$$

$$x=\frac{-R(\mathrm{A}+1)c^{*}+R}{-R+\mathrm{A}((R\beta)^{1/\rho}-R)+1}+1$$

Because of $\left(\frac{(R\beta)^{-1}-1+\theta}{\lambda\theta}\right)^{1/\rho}=\mathrm{A}$, the final formula is

$$x = \frac{-R\left(\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)^{1/\rho} + 1\right)c^* + R}{-R + \left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)^{1/\rho}((R\beta)^{1/\rho} - R) + 1} + 1$$

When $\rho = 1$,

$$x = \frac{-R\left(\left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right) + 1\right)c^* + R}{-R + \left(\frac{(R\beta)^{-1} - 1 + \theta}{\lambda\theta}\right)(R\beta - R) + 1} + 1$$

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